

# Elasticity

1

- 1 A glass fibre of length 0.24 m and area of cross-section  $7.9 \times 10^{-7} \text{ m}^2$  is tested until it breaks. The variation with load  $F$  of the extension  $x$  of the fibre is shown in Fig. 4.1.

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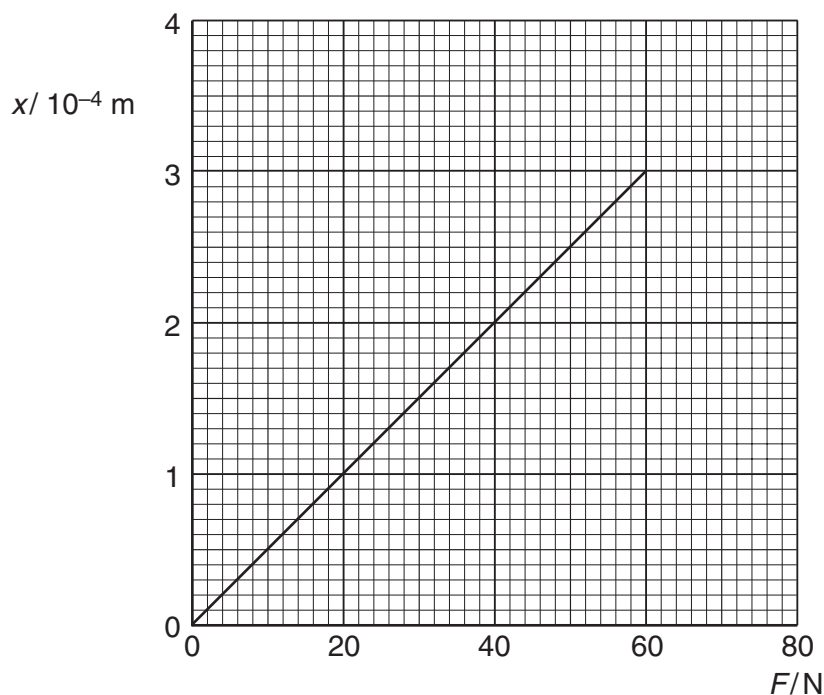


Fig. 4.1

- (a) State whether glass is ductile, brittle or polymeric.

.....[1]

- (b) Use Fig. 4.1 to determine, for this sample of glass,

- (i) the ultimate tensile stress,

ultimate tensile stress = ..... Pa [2]

(ii) the Young modulus,

Young modulus = ..... Pa [3]

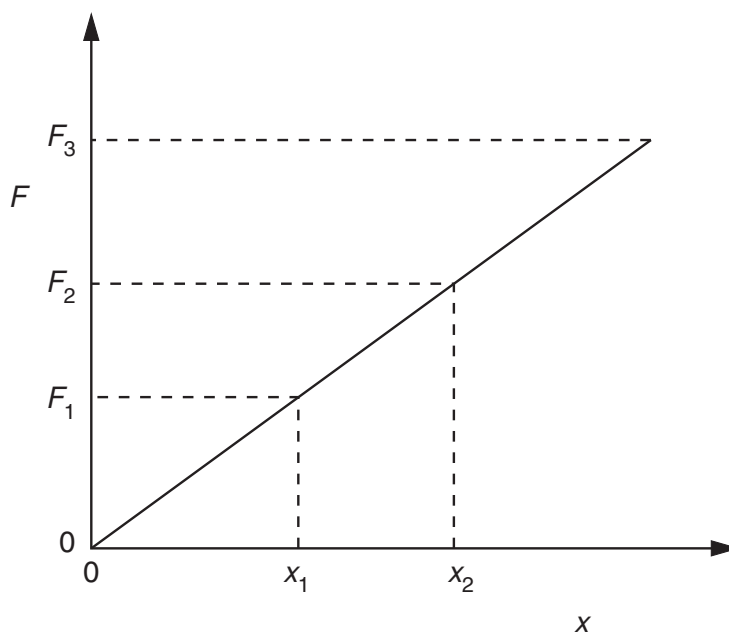
(iii) the maximum strain energy stored in the fibre before it breaks.

maximum strain energy = ..... J [2]

(c) A hard ball and a soft ball, with equal masses and volumes, are thrown at a glass window. The balls hit the window at the same speed. Suggest why the hard ball is more likely than the soft ball to break the glass window.

.....  
.....  
.....  
.....[3]

- 2 Fig. 5.1 shows the variation with force  $F$  of the extension  $x$  of a spring as the force is increased to  $F_3$  and then decreased to zero.



**Fig. 5.1**

- (a) State, with a reason, whether the spring is undergoing an elastic change.

.....  
 ..... [1]

- (b) The extension of the spring is increased from  $x_1$  to  $x_2$ .

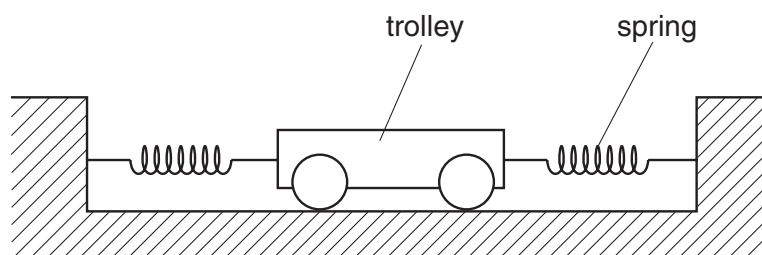
Show that the work  $W$  done in extending the spring is given by

$$W = \frac{1}{2}k(x_2^2 - x_1^2),$$

where  $k$  is the spring constant.

[3]

- (c) A trolley of mass 850 g is held between two fixed points by means of identical springs, as shown in Fig. 5.2.



**Fig. 5.2**

When the trolley is in equilibrium, the springs are each extended by 4.5 cm. Each spring has a spring constant  $16 \text{ N cm}^{-1}$ .

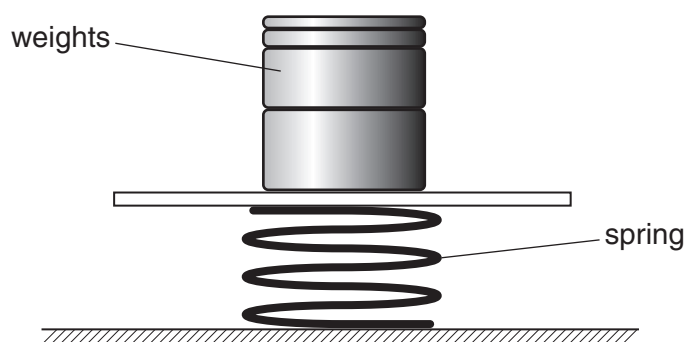
The trolley is moved a distance of 1.5 cm along the direction of the springs. This causes the extension of one spring to be increased and the extension of the other spring to be decreased. The trolley is then released. The trolley accelerates and reaches its maximum speed at the equilibrium position.

Assuming that the springs obey Hooke's law, use the expression in (b) to determine the maximum speed of the trolley.

speed = .....  $\text{m s}^{-1}$  [4]

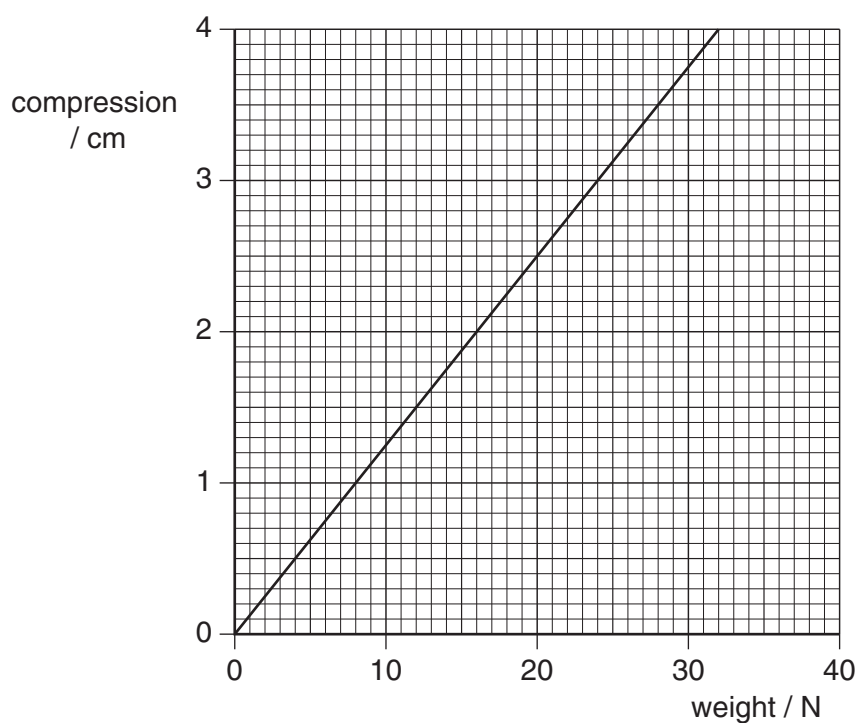
- 3 A spring is placed on a flat surface and different weights are placed on it, as shown in Fig. 2.1.

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**Fig. 2.1**

The variation with weight of the compression of the spring is shown in Fig. 2.2.



**Fig. 2.2**

The elastic limit of the spring has not been exceeded.

- (a) (i) Determine the spring constant  $k$  of the spring.

$k = \dots\dots\dots \text{Nm}^{-1}$  [2]

- (ii) Deduce that the strain energy stored in the spring is 0.49 J for a compression of 3.5 cm.

[2]

- (b) Two trolleys, of masses 800 g and 2400 g, are free to move on a horizontal table. The spring in (a) is placed between the trolleys and the trolleys are tied together using thread so that the compression of the spring is 3.5 cm, as shown in Fig. 2.3.

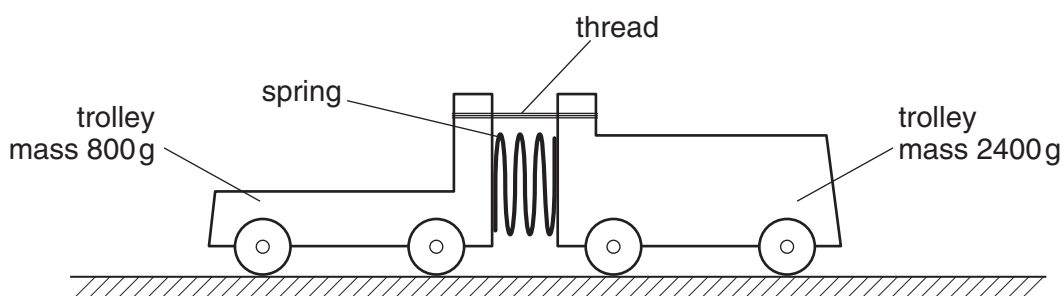


Fig. 2.3

Initially, the trolleys are not moving.  
The thread is then cut and the trolleys move apart.

- (i) Deduce that the ratio

$$\frac{\text{speed of trolley of mass 800 g}}{\text{speed of trolley of mass 2400 g}}$$

is equal to 3.0.

[2]

- (ii) Use the answers in **(a)(ii)** and **(b)(i)** to calculate the speed of the trolley of mass 800 g.

For  
Examiner's  
Use

speed = .....  $\text{ms}^{-1}$  [3]

- 4 A spring having spring constant  $k$  hangs vertically from a fixed point. A load of weight  $L$ , when hung from the spring, causes an extension  $e$ . The elastic limit of the spring is not exceeded.

(a) State

- (i) what is meant by an *elastic deformation*,

.....  
.....  
..... [2]

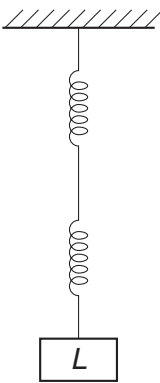
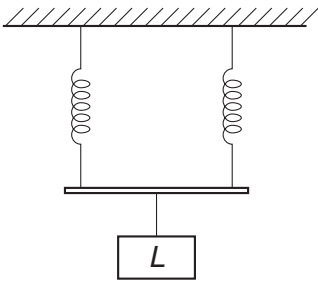
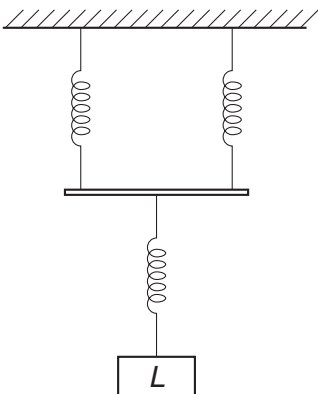
- (ii) the relation between  $k$ ,  $L$  and  $e$ .

..... [1]

Question 4 continues on page 10



(b) Some identical springs, each with spring constant  $k$ , are arranged as shown in Fig. 4.1.

arrangement	total extension	spring constant of arrangement
	.....	.....
	.....	.....
	.....	.....

**Fig. 4.1**

The load on each of the arrangements is  $L$ .

For each arrangement in Fig. 4.1, complete the table by determining

- (i) the total extension in terms of  $e$ ,
- (ii) the spring constant in terms of  $k$ .

[5]

- 5 (a) In the following list of solids, underline those materials which are crystalline.

rubber      copper      nylon      glass      aluminium      [2]

- (b) The three graphs A, B and C of Fig. 5.1 represent the variation with extension  $x$  of the tension  $F$  in specimens of three different materials. One of the materials is polymeric, one is brittle and the other is ductile. They are not shown in that order in Fig. 5.1.

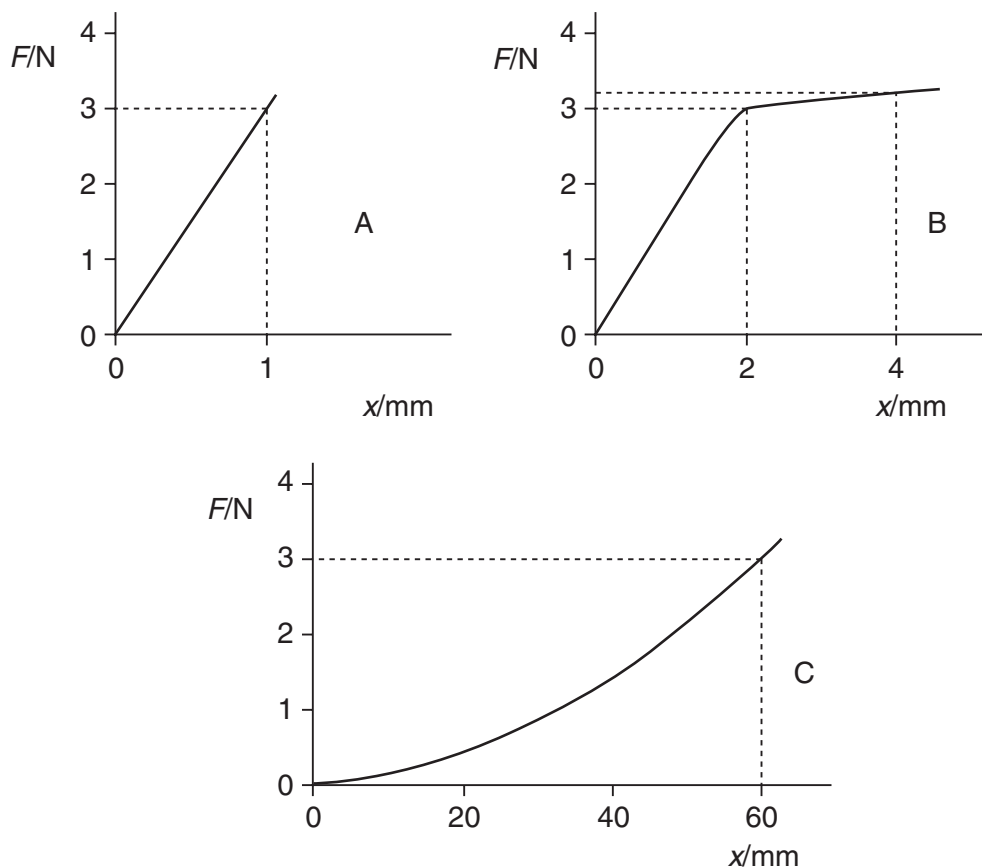


Fig. 5.1

- (i) State the type of material which would produce the line shown in each graph.

Graph A is for a ..... material.

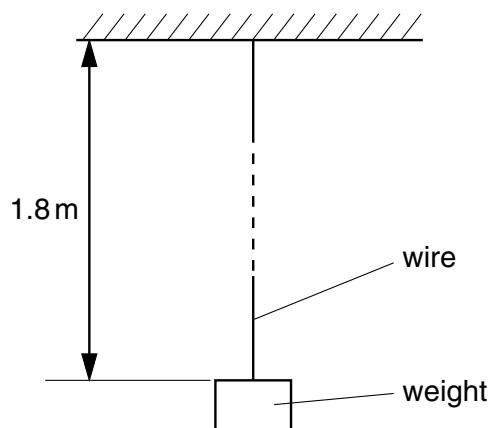
Graph B is for a ..... material.

Graph C is for a ..... material. [2]

- (ii) Use graph B to estimate the work done in stretching the specimen from 0 to 4 mm.

work done = ..... J [3]

- 6 An aluminium wire of length 1.8 m and area of cross-section  $1.7 \times 10^{-6} \text{ m}^2$  has one end fixed to a rigid support. A small weight hangs from the free end, as illustrated in Fig. 9.1.



**Fig. 9.1**

The resistance of the wire is  $0.030 \, \Omega$  and the Young modulus of aluminium is  $7.1 \times 10^{10} \text{ Pa}$ .

The load on the wire is increased by 25 N.

**(a)** Calculate

**(i)** the increase in stress,

increase = ..... Pa

**(ii)** the change in length of the wire.

change = ..... m  
[4]

- (b) Assuming that the area of cross-section of the wire does not change when the load is increased, determine the change in resistance of the wire.

change = .....  $\Omega$  [3]

- 7 (a) A metal wire has an unstretched length  $L$  and area of cross-section  $A$ . When the wire supports a load  $F$ , the wire extends by an amount  $\Delta L$ . The wire obeys Hooke's law.

Write down expressions, in terms of  $L$ ,  $A$ ,  $F$  and  $\Delta L$ , for

- (i) the applied stress,

.....

- (ii) the tensile strain in the wire,

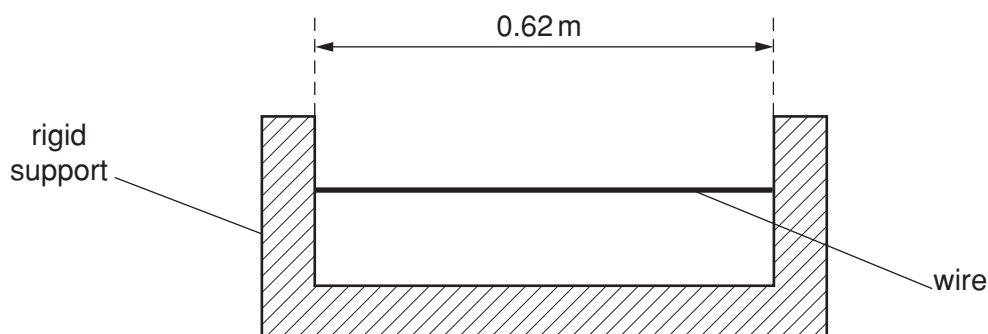
.....

- (iii) the Young modulus of the material of the wire.

.....

[3]

- (b) A steel wire of uniform cross-sectional area  $7.9 \times 10^{-7} \text{ m}^2$  is heated to a temperature of 650 K. It is then clamped between two rigid supports, as shown in Fig. 5.1.



**Fig. 5.1**

The wire is straight but not under tension and the length between the supports is 0.62 m. The wire is then allowed to cool to 300 K.

When the wire is allowed to contract freely, a 1.00 m length of the wire decreases in length by 0.012 mm for every 1 K decrease in temperature.

- (i) Show that the change in length of the wire, if it were allowed to contract as it cools from 650 K to 300 K, would be 2.6 mm.

[2]

- (ii) The Young modulus of steel is  $2.0 \times 10^{11}$  Pa. Calculate the tension in the wire at 300 K, assuming that the wire obeys Hooke's law.

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tension = ..... N [2]

- (iii) The ultimate tensile stress of steel is 250 MPa. Use this information and your answer in (ii) to suggest whether the wire will, in practice, break as it cools.

.....  
..... [3]

- 7 A straight wire of unstretched length  $L$  has an electrical resistance  $R$ . When it is stretched by a force  $F$ , the wire extends by an amount  $\Delta L$  and the resistance increases by  $\Delta R$ . The area of cross-section  $A$  of the wire may be assumed to remain constant.

(a) (i) State the relation between  $R$ ,  $L$ ,  $A$  and the resistivity  $\rho$  of the material of the wire.

.....  
 ..... [1]

(ii) Show that the fractional change in resistance  $\frac{\Delta R}{R}$  is equal to the strain in the wire.

[2]

(b) A steel wire has area of cross-section  $1.20 \times 10^{-7} \text{ m}^2$  and a resistance of  $4.17 \Omega$ .

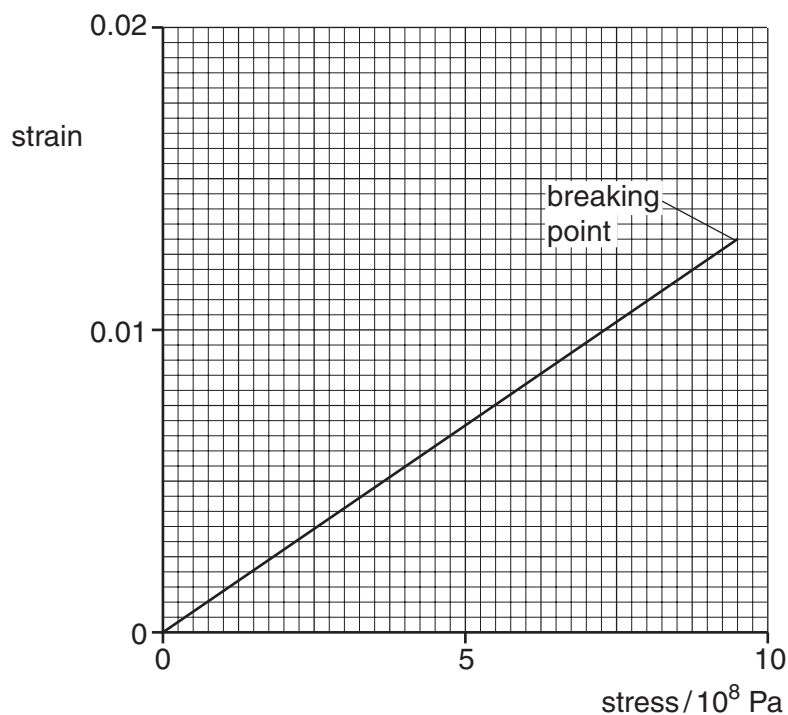
The Young modulus of steel is  $2.10 \times 10^{11} \text{ Pa}$ .

The tension in the wire is increased from zero to  $72.0 \text{ N}$ . The wire obeys Hooke's law at these values of tension.

Determine the strain in the wire and hence its change in resistance. Express your answer to an appropriate number of significant figures.

change = .....  $\Omega$  [5]

- 8 A sample of material in the form of a cylindrical rod has length  $L$  and uniform area of cross-section  $A$ . The rod undergoes an increasing tensile stress until it breaks. Fig. 4.1 shows the variation with stress of the strain in the rod.



**Fig. 4.1**

- (a) State whether the material of the rod is ductile, brittle or polymeric.

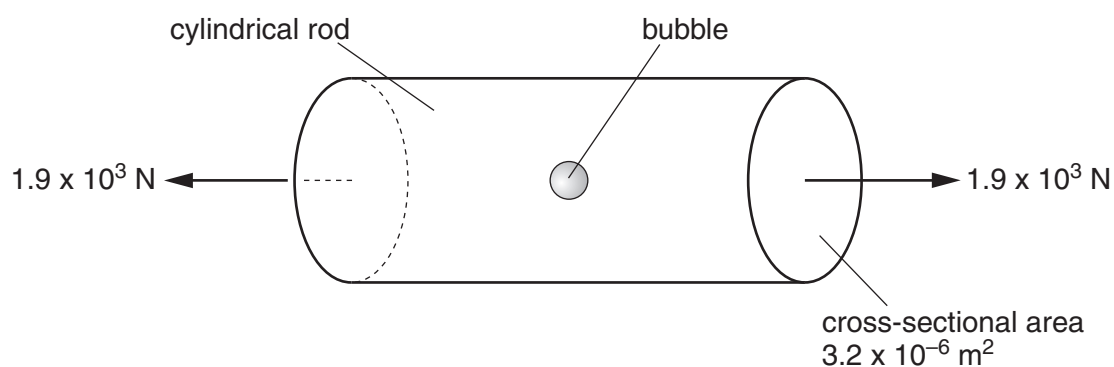
..... [1]

- (b) Determine the Young modulus of the material of the rod.

Young modulus = ..... Pa [2]



- (c) A second cylindrical rod of the same material has a spherical bubble in it, as illustrated in Fig. 4.2.



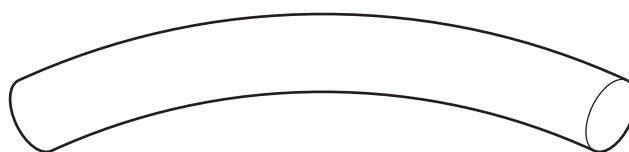
**Fig. 4.2**

The rod has an area of cross-section of  $3.2 \times 10^{-6} \text{ m}^2$  and is stretched by forces of magnitude  $1.9 \times 10^3 \text{ N}$ .

By reference to Fig. 4.1, calculate the maximum area of cross-section of the bubble such that the rod does not break.

area = .....  $\text{m}^2$  [3]

- (d) A straight rod of the same material is bent as shown in Fig. 4.3.



**Fig. 4.3**

Suggest why a thin rod can bend more than a thick rod without breaking.

.....  
 .....  
 ..... [2]

- 9 A uniform wire has length  $L$  and area of cross-section  $A$ . The wire is fixed at one end so that it hangs vertically with a load attached to its free end, as shown in Fig. 4.1.

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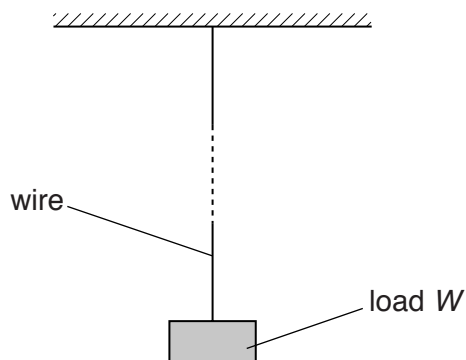


Fig. 4.1

When the load of magnitude  $W$  is attached to the wire, it extends by an amount  $e$ . The elastic limit of the wire is not exceeded.

The material of the wire has resistivity  $\rho$ .

- (a) (i) Explain what is meant by extends *elastically*.

.....  
 .....  
 ..... [2]

- (ii) Write down expressions, in terms of  $L$ ,  $A$ ,  $W$ ,  $\rho$  and  $e$  for

1. the resistance  $R$  of the unstretched wire,

$$R = \dots\dots\dots [1]$$

2. the Young modulus  $E$  of the wire.

$$E = \dots\dots\dots [1]$$

**(b)** A steel wire has resistance  $0.44\ \Omega$ . Steel has resistivity  $9.2 \times 10^{-8}\ \Omega\text{m}$ .

A load of  $34\text{ N}$  hung from the end of the wire causes an extension of  $7.7 \times 10^{-4}\text{ m}$ .

Using your answers in **(a)(ii)**, calculate the Young modulus  $E$  of steel.

*For  
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Use*

$E = \dots\dots\dots\text{ Pa [3]}$

- 10 (a) Explain what is meant by *strain energy* (*elastic potential energy*).

For  
Examiner's  
Use

.....

.....

..... [2]

- (b) A spring that obeys Hooke's law has a spring constant  $k$ .

Show that the energy  $E$  stored in the spring when it has been extended elastically by an amount  $x$  is given by

$$E = \frac{1}{2}kx^2.$$

[3]

- (c) A light spring of unextended length 14.2 cm is suspended vertically from a fixed point, as illustrated in Fig. 4.1.

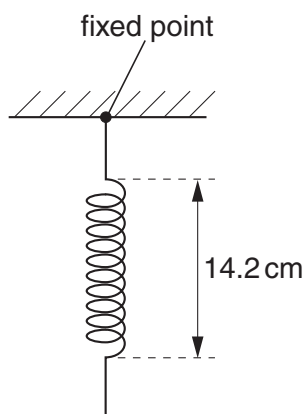


Fig. 4.1

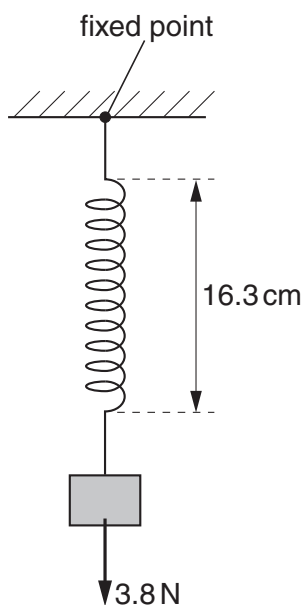


Fig. 4.2

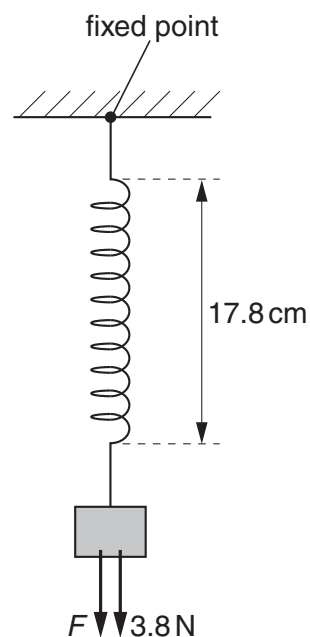


Fig. 4.3

A mass of weight 3.8 N is hung from the end of the spring, as shown in Fig. 4.2. The length of the spring is now 16.3 cm.

An additional force  $F$  then extends the spring so that its length becomes 17.8 cm, as shown in Fig. 4.3.

The spring obeys Hooke's law and the elastic limit of the spring is not exceeded.

- (i) Show that the spring constant of the spring is  $1.8 \text{ N cm}^{-1}$ .

[1]

(ii) For the extension of the spring from a length of 16.3 cm to a length of 17.8 cm,

1. calculate the change in the gravitational potential energy of the mass on the spring,

change in energy = ..... J [2]

2. show that the change in elastic potential energy of the spring is 0.077 J,

[1]

3. determine the work done by the force  $F$ .

work done = ..... J [1]

**11 (a) (i) Define the terms****1. tensile stress,**

.....

.....[1]

**2. tensile strain,**

.....

.....[1]

**3. the Young modulus.**

.....

.....[1]

**(ii) Suggest why the Young modulus is not used to describe the deformation of a liquid or a gas.**

.....

.....[1]

**(b) The change  $\Delta V$  in the volume  $V$  of some water when the pressure on the water increases by  $\Delta p$  is given by the expression**

$$\Delta p = 2.2 \times 10^9 \frac{\Delta V}{V},$$

where  $\Delta p$  is measured in pascal.

In many applications, water is assumed to be incompressible.

By reference to the expression, justify this assumption.

.....

.....

.....[2]

(c) Normal atmospheric pressure is  $1.01 \times 10^5$  Pa.

Divers in water of density  $1.08 \times 10^3 \text{ kg m}^{-3}$  frequently use an approximation that every 10 m increase in depth of water is equivalent to one atmosphere increase in pressure. Determine the percentage error in this approximation.

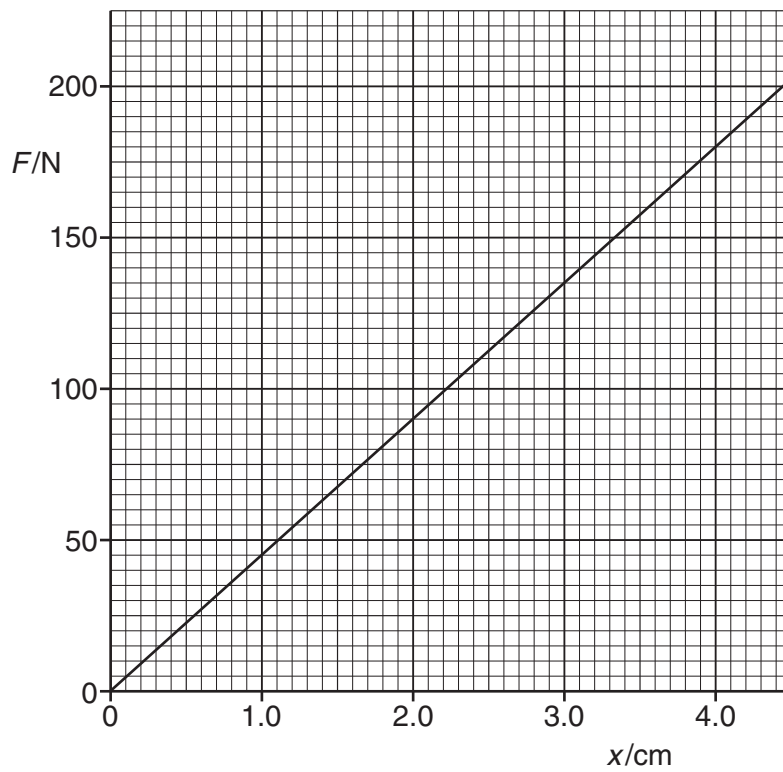
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error = ..... % [3]



- 3 (a) The variation with extension  $x$  of the tension  $F$  in a spring is shown in Fig. 3.1.

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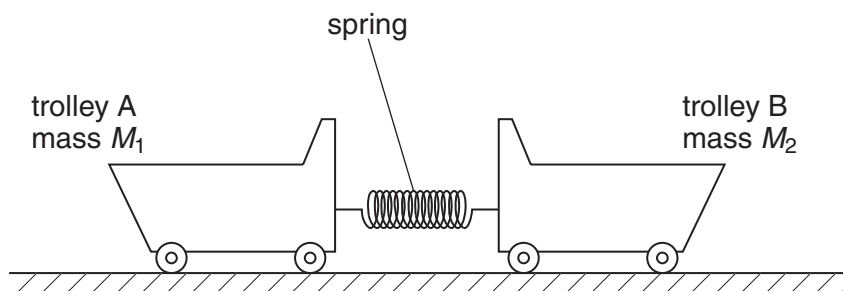
**Fig. 3.1**

Use Fig. 3.1 to calculate the energy stored in the spring for an extension of 4.0 cm.  
Explain your working.

energy = ..... J [3]

- (b) The spring in (a) is used to join together two frictionless trolleys A and B of mass  $M_1$  and  $M_2$  respectively, as shown in Fig. 3.2.

For  
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**Fig. 3.2**

The trolleys rest on a horizontal surface and are held apart so that the spring is extended.

The trolleys are then released.

- (i) Explain why, as the extension of the spring is reduced, the momentum of trolley A is equal in magnitude but opposite in direction to the momentum of trolley B.

.....  
 .....  
 .....  
 ..... [2]

- (ii) At the instant when the extension of the spring is zero, trolley A has speed  $V_1$  and trolley B has speed  $V_2$ .

Write down

1. an equation, based on momentum, to relate  $V_1$  and  $V_2$ ,

.....  
 ..... [1]

2. an equation to relate the initial energy  $E$  stored in the spring to the final energies of the trolleys.

.....  
 ..... [1]

- (iii) 1. Show that the kinetic energy  $E_k$  of an object of mass  $m$  is related to its momentum  $p$  by the expression

$$E_k = \frac{p^2}{2m}.$$

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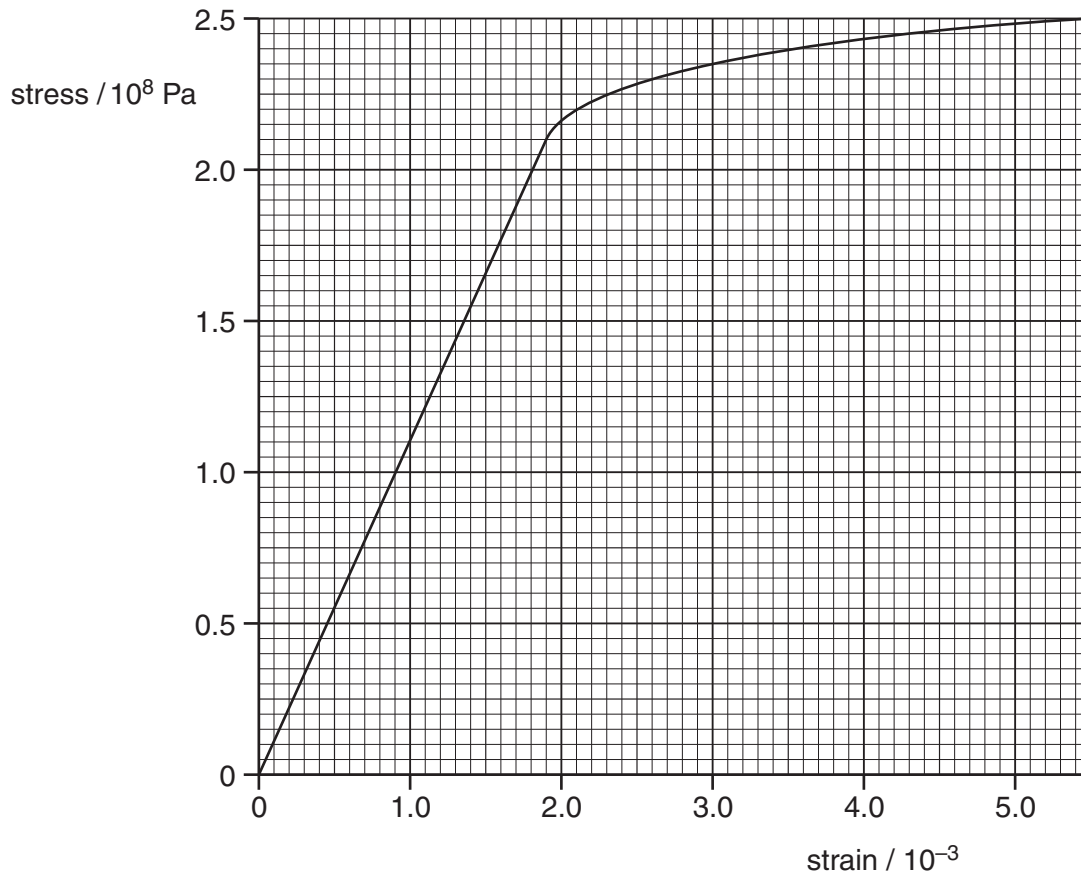
[1]

2. Trolley A has a larger mass than trolley B.  
Use your answer in **(ii) part 1** to deduce which trolley, A or B, has the larger kinetic energy at the instant when the extension of the spring is zero.

.....  
.....[1]

- 5 (a) Tensile forces are applied to opposite ends of a copper rod so that the rod is stretched. The variation with stress of the strain of the rod is shown in Fig. 5.1.

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**Fig. 5.1**

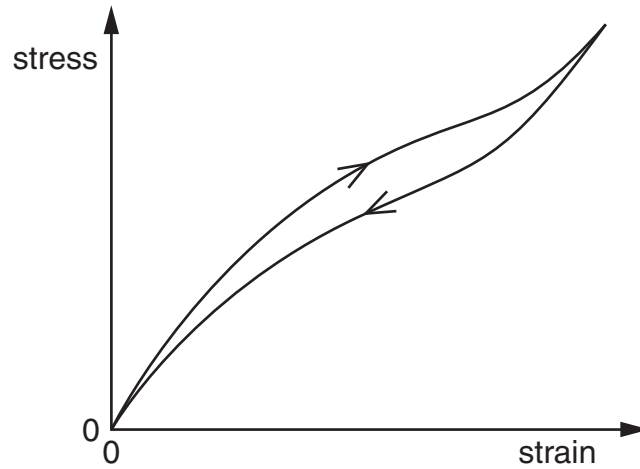
- (i) Use Fig. 5.1 to determine the Young modulus of copper.

Young modulus = ..... Pa [3]

- (ii) On Fig. 5.1, sketch a line to show the variation with stress of the strain of the rod as the stress is reduced from  $2.5 \times 10^6$  Pa to zero. No further calculations are expected. [1]

- (b) The walls of the tyres on a car are made of a rubber compound. The variation with stress of the strain of a specimen of this rubber compound is shown in Fig. 5.2.

For  
Examiner's  
Use



**Fig. 5.2**

As the car moves, the walls of the tyres bend and straighten continuously.

Use Fig. 5.2 to explain why the walls of the tyres become warm.

.....

.....

.....

.....[3]

- 4 (a) A uniform wire has length  $L$  and constant area of cross-section  $A$ . The material of the wire has Young modulus  $E$  and resistivity  $\rho$ . A tension  $F$  in the wire causes its length to increase by  $\Delta L$ .

For this wire, state expressions, in terms of  $L$ ,  $A$ ,  $F$ ,  $\Delta L$  and  $\rho$  for

- (i) the stress  $\sigma$ ,

..... [1]

- (ii) the strain  $\epsilon$ ,

..... [1]

- (iii) the Young modulus  $E$ ,

..... [1]

- (iv) the resistance  $R$ .

..... [1]

- (b) One end of a metal wire of length 2.6 m and constant area of cross-section  $3.8 \times 10^{-7} \text{ m}^2$  is attached to a fixed point, as shown in Fig. 4.1.

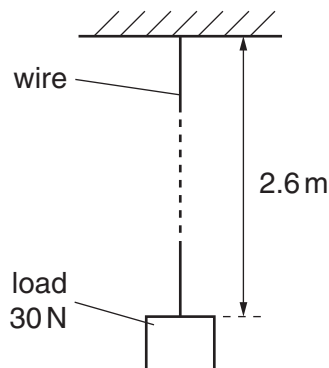


Fig. 4.1

The Young modulus of the material of the wire is  $7.0 \times 10^{10} \text{ Pa}$  and its resistivity is  $2.6 \times 10^{-8} \Omega \text{ m}$ .

A load of 30 N is attached to the lower end of the wire. Assume that the area of cross-section of the wire does not change.

For this load of 30 N,

- (i) show that the extension of the wire is 2.9 mm,

[1]

- (ii) calculate the change in resistance of the wire.

change = .....  $\Omega$  [2]

- (c) The resistance of the wire changes with the applied load.  
Comment on the suggestion that this change of resistance could be used to measure the magnitude of the load on the wire.

.....  
 .....  
 ..... [2]

40

- 4 (a) A metal wire has spring constant  $k$ . Forces are applied to the ends of the wire to extend it within the limit of Hooke's law.  
Show that, for an extension  $x$ , the strain energy  $E$  stored in the wire is given by

$$E = \frac{1}{2}kx^2.$$

For  
Examiner's  
Use

[4]

- (b) The wire in (a) is now extended beyond its elastic limit. The forces causing the extension are then removed.

The variation with extension  $x$  of the tension  $F$  in the wire is shown in Fig. 4.1.

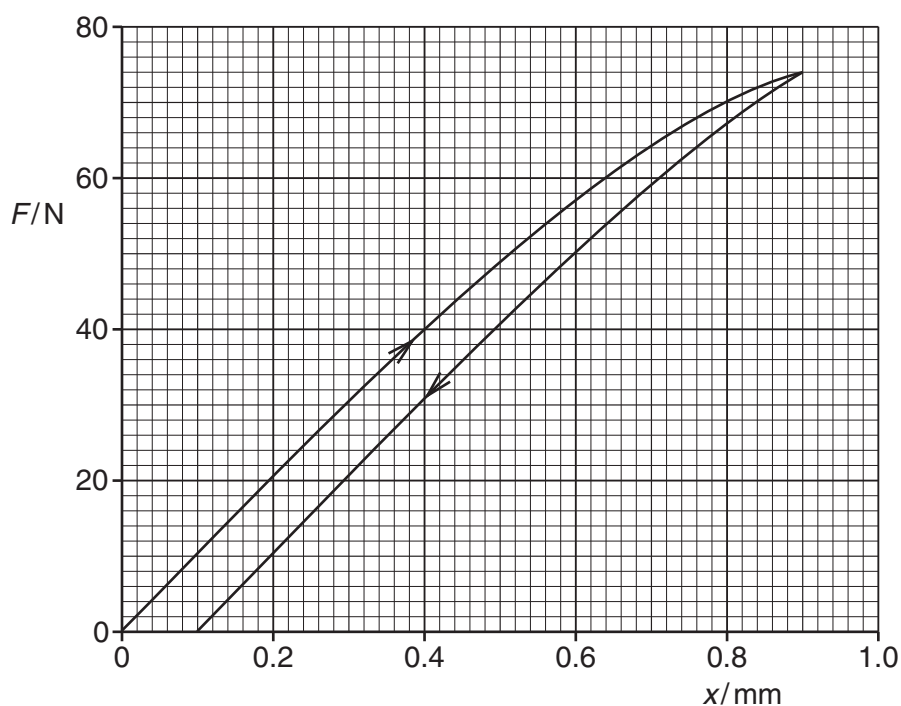


Fig. 4.1

Energy  $E_S$  is expended to cause a permanent extension of the wire.

- (i) On Fig. 4.1, shade the area that represents the energy  $E_S$ .

[1]



- (ii) Use Fig. 4.1 to calculate the energy  $E_S$ .

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$$E_S = \dots\dots\dots \text{ mJ [3]}$$

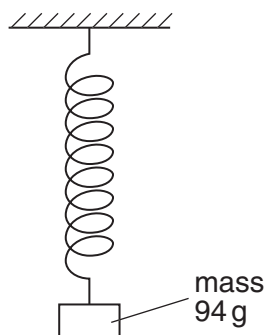
- (iii) Suggest the change in the structure of the wire that is caused by the energy  $E_S$ .

.....

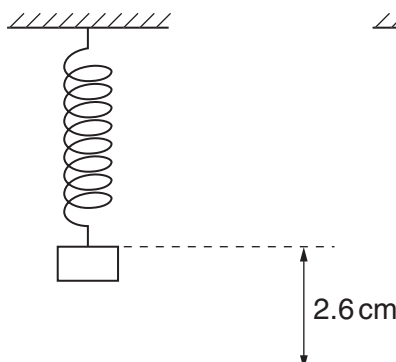
..... [1]

- 5 A spring hangs vertically from a fixed point and a mass of 94 g is suspended from the spring, stretching the spring as shown in Fig. 5.1.

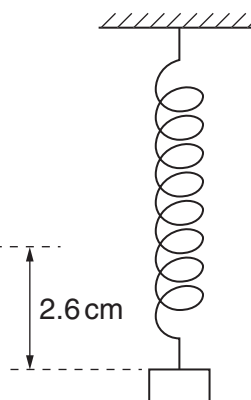
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**Fig. 5.1**



**Fig. 5.2**



**Fig. 5.3**

The mass is raised vertically so that the length of the spring is its unextended length. This is illustrated in Fig. 5.2.

The mass is then released. The mass moves through a vertical distance of 2.6 cm before temporarily coming to rest. This position is illustrated in Fig. 5.3.

- (a) State which diagram, Fig. 5.1, Fig. 5.2 or Fig. 5.3, illustrates the position of the mass such that

- (i) the mass has maximum gravitational potential energy,

..... [1]

- (ii) the spring has maximum strain energy.

..... [1]

- (b) Briefly describe the variation of the kinetic energy of the mass as the mass falls from its highest position (Fig. 5.2) to its lowest position (Fig. 5.3).

.....

..... [1]

- (c) The strain energy  $E$  stored in the spring is given by the expression

$$E = \frac{1}{2}kx^2$$

where  $k$  is the spring constant and  $x$  is the extension of the spring.

For the mass moving between the positions shown in Fig. 5.2 and Fig. 5.3,

- (i) calculate the change in the gravitational potential energy of the mass,

change = ..... J [2]

- (ii) determine the extension of the spring at which the strain energy is half its maximum value.

extension = ..... cm [3]

For  
Examiner's  
Use

4 A student measures the Young modulus of a metal in the form of a wire.

For  
Examiner's  
Use

(a) Describe, with the aid of a diagram, the apparatus that could be used.

.....

.....

.....

..... [2]

(b) Describe the method used to obtain the required measurements.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

..... [4]

- (c) Describe how the measurements taken can be used to determine the Young modulus.

For  
Examiner's  
Use

.....

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.....

.....

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.....

..... [4]

- 4 (a) State Hooke's Law.

For  
Examiner's  
Use

.....  
..... [1]

- (b) A spring is compressed by applying a force. The variation with compression  $x$  of the force  $F$  is shown in Fig. 4.1.

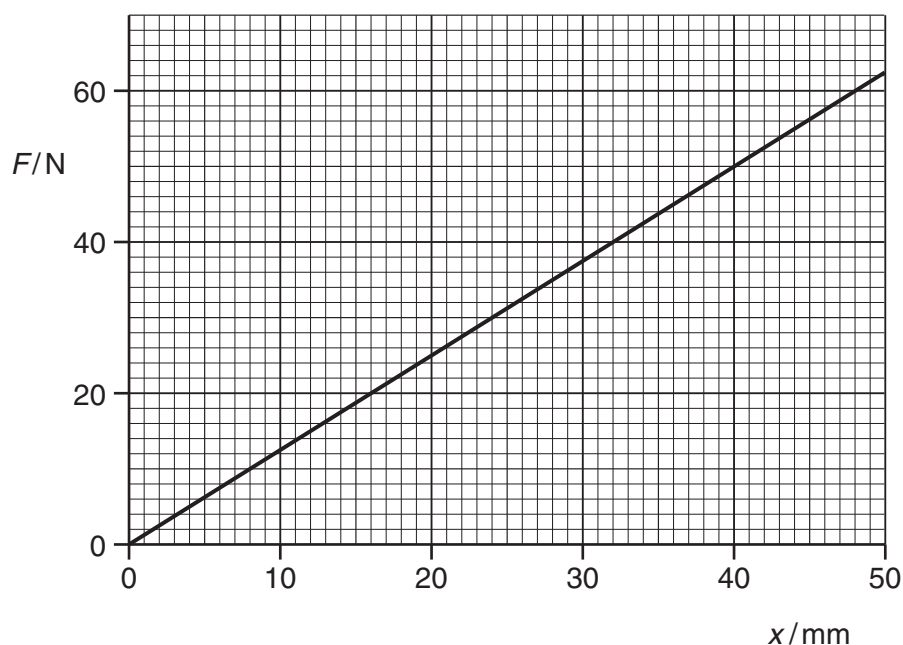


Fig. 4.1

- (i) Calculate the spring constant.

spring constant = .....  $\text{Nm}^{-1}$  [1]

- (ii) Show that the work done in compressing the spring by 36 mm is 0.81 J.

[2]

- (c) A child's toy uses the spring in (b) to shoot a small ball vertically upwards. The ball has a mass of 25 g. The toy is shown in Fig. 4.2.

For  
Examiner's  
Use

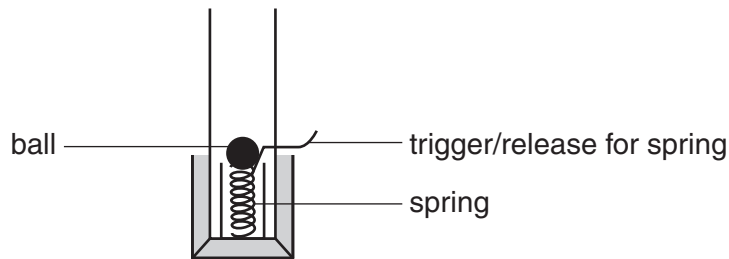


Fig. 4.2

- (i) The spring in the toy is compressed by 36 mm. The spring is released. Assume all the strain energy in the spring is converted to kinetic energy of the ball. Using the result in (b)(ii), calculate the speed with which the ball leaves the spring.

speed = .....  $\text{ms}^{-1}$  [2]

- (ii) Determine the compression of the spring required for the ball to leave the spring with twice the speed determined in (i).

compression = ..... mm [2]

- (iii) Determine the ratio

$$\frac{\text{maximum possible height for compression in (i)}}{\text{maximum possible height for compression in (ii)}}$$

ratio = ..... [2]

4 (a) Define, for a wire,

(i) *stress*,

.....  
 ..... [1]

(ii) *strain*.

.....  
 ..... [1]

(b) A wire of length 1.70 m hangs vertically from a fixed point, as shown in Fig. 4.1.

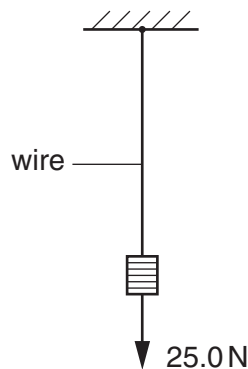


Fig. 4.1

The wire has cross-sectional area  $5.74 \times 10^{-8} \text{ m}^2$  and is made of a material that has a Young modulus of  $1.60 \times 10^{11} \text{ Pa}$ . A load of 25.0 N is hung from the wire.

(i) Calculate the extension of the wire.

extension = ..... m [3]

(ii) The same load is hung from a second wire of the same material. This wire is twice the length but the **same volume** as the first wire. State and explain how the extension of the second wire compares with that of the first wire.

.....  
 .....  
 ..... [3]