## Math Classified According to the Syllabus IGCSE (0580)

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## EXTENDED MATHEMATICS 2002-2011 CLASSIFIEDS FUNCTIONSEGRAPHS

## Compiled \& Edited

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$$
\begin{gathered}
\mathrm{f}(x)=\frac{1}{x+4} \quad(x \neq-4) \\
\mathrm{g}(x)=x^{2}-3 x \\
\mathrm{~h}(x)=x^{3}+1
\end{gathered}
$$

(a) Work out fg(1).
(b) Find $\mathrm{h}^{-1}(x)$.

(c) Solve the equation $\mathrm{g}(x)=-2$.

5 (a) Complete the table for the function $\mathrm{f}(x)=\frac{x^{3}}{2}-3 x-1$.

| $x$ | -3 | -2 | -1.5 | -1 | 0 | 1 | 1.5 | 2 | 3 | 3.5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | -5.5 |  | 1.8 | 1.5 |  | -3.5 | -3.8 | -3 |  | 9.9 |

(b) On the grid draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant 3.5$.

(c) Use your graph to
(i) solve $\mathrm{f}(x)=0.5$,

$$
\operatorname{Answer}(c)(\text { i) } x=\text {............. or } x=\ldots, \ldots . . . .
$$

(ii) find the inequalities for $k$, so that $\mathrm{f}(x)=k$ has only 1 answer.

$$
\begin{aligned}
\text { Answer(c)(ii) } k & <. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~ \\
k & \\
& \\
& . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~
\end{aligned}
$$

(d) (i) On the same grid, draw the graph of $y=3 x-2$ for $-1 \leqslant x \leqslant 3.5$.
(ii) The equation $\frac{x^{3}}{2}-3 x-1=3 x-2$ can be written in the form $x^{3}+a x+b=0$. Find the values of $a$ and $b$.

(iii) Use your graph to find the positive answers to $\frac{x^{3}}{2}-3 x-1=3 x-2$ for $-3 \leqslant x \leqslant 3.5$.

$$
\begin{aligned}
& \mathrm{f}(x)=4 x-2 \\
& \mathrm{~g}(x)=\frac{2}{x}+1 \\
& \mathrm{~h}(x)=x^{2}+3
\end{aligned}
$$

(a) (i) Find the value of $\mathrm{hf}(2)$.
Answer(a)(i)
(ii) Write $\operatorname{fg}(x)$ in its simplest form.
(b) Solve $\mathrm{g}(x)=0.2$.


Answer(b) $x=$
[2]
(c) Find the value of $\operatorname{gg}(3)$.
(d) (i) Show that $\mathrm{f}(x)=\mathrm{g}(x)$ can be written as $4 x^{2}-3 x-2=0$. Answer (d)(i)
(ii) Solve the equation $4 x^{2}-3 x-2=0$.

Show all your working and give your answers correct to 2 decimal places.


Answer(d)(ii) $x=$
or $x=$
[4]

7 The diagram shows the accurate graph of $y=\mathrm{f}(x)$ where $\mathrm{f}(x)=\frac{1}{x}+x^{2}$ for $0<x \leqslant 3$.

(a) Complete the table for $\mathrm{f}(x)=\frac{1}{x}+x^{2}$.

| $x$ | -3 | -2 | -1 | -0.5 | -0.3 | -0.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ |  | 3.5 | 0 | -1.8 |  |  |

(b) On the grid, draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x<0$.
(c) By drawing a tangent, work out an estimate of the gradient of the graph where $x=2$.
(d) Write down the inequality satisfied by $k$ when $\mathrm{f}(x)=k$ has three answers.
(e) (i) Draw the line $y=1-x$ on the grid for $-3 \leqslant x \leqslant 3$.
(ii) Use your graphs to solve the equation $1-x=\frac{1}{x}+x^{2}$.

$$
\text { Answer(e)(ii) } x=
$$

(f) (i) Rearrange $x^{3}-x^{2}-2 x+1=0$ into the form $\frac{1}{x}+x^{2}=a x+b$, where $a$ and $b$ are integers. Answer(f)(i)
(ii) Write down the equation of the line that could be drawn on the graph to solve $x^{3}-x^{2}-2 x+1=0$.

$$
\text { Answer }(f)(\mathrm{ii)} y=
$$

2 (a) Complete the table of values for $y=2^{x}$.

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 0.25 |  | 1 | 2 |  | 8 |

(b) On the grid, draw the graph of $y=2^{x}$ for $-2 \leqslant x \leqslant 3$.

(c) (i) On the grid, draw the straight line which passes through the points $(0,2)$ and $(3,8)$.
(ii) The equation of this line is $y=m x+2$.

Show that the value of $m$ is 2 .
Answer(c)(ii)
(iii) One answer to the equation $2^{x}=2 x+2$ is $x=3$.

Use your graph to find the other answer.
(d) Draw the tangent to the curve at the point where $x=1$.

Use this tangent to calculate an estimate of the gradient of $y=2^{x}$ when $x=1$.
$8 \mathrm{f}(x)=x^{2}+x-1$
$g(x)=1-2 x$
$h(x)=3^{x}$
(a) Find the value of $\operatorname{hg}(-2)$.

> Answer(a)
(b) Find $\mathrm{g}^{-1}(x)$.
$\operatorname{Answer}(b) \mathrm{g}^{-1}(x)=$
(c) Solve the equation $\mathrm{f}(x)=0$.

Show all your working and give your answers correct to 2 decimal places.

(d) Find $\mathrm{fg}(x)$.

Give your answer in its simplest form.

$$
\operatorname{Answer}(d) \operatorname{fg}(x)=
$$

(e) Solve the equation $\mathrm{h}^{-1}(x)=2$.

$$
\text { Answer(e) } x=
$$

(b) Using a scale of 2 cm to represent 1 minute on the horizontal $t$-axis and 2 cm to represent 10 metres on the vertical $d$-axis, draw the graph of $\quad d=(t+1)^{2}+\frac{48}{(t+1)}-20 \quad$ for $0 \leqslant \mathrm{t} \leqslant 7$.
(c) Mark and label $F$ the point on your graph when the fish is 12 metres from Dimitra and swimming away from her. Write down the value of $t$ at this point, correct to one decimal place.
(d) For how many minutes is the fish less than 10 metres from Dimitra?
(e) By drawing a suitable line on your grid, calculate the speed of the fish when $t=2.5$.


20 (a) Complete the table of values for $y=3^{x}$.

| $x$ | -2 | -1.5 | -1 | -0.5 | 0 | 0.5 | 1 | 1.5 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ |  | 0.2 |  |  |  |  |  | 5.2 | 9 |

(b) Use your table to complete the graph of $y=3^{x}$ for $-2 \leqslant x \leqslant 2$.

[2]
(c) Use the graph to find the solution of the equation

$$
3^{x}=6
$$

Answer (c) $x=$

4 Answer the whole of this question on a sheet of graph paper.

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| $\mathrm{f}(x)$ | -8 | 4.5 | 8 | 5.5 | 0 | -5.5 | -8 | -4.5 | 8 |

(a) Using a scale of 2 cm to represent 1 unit on the $x$-axis and 2 cm to represent 4 units on the $y$-axis, draw axes for $-4 \leqslant x \leqslant 4$ and $-8 \leqslant y \leqslant 8$.
Draw the curve $y=\mathrm{f}(x)$ using the table of values given above.
(b) Use your graph to solve the equation $\mathrm{f}(x)=0$.
(c) On the same grid, draw $y=\mathrm{g}(x)$ for $-4 \leqslant x \leqslant 4$, where $\mathrm{g}(x)=x+1$.
(d) Write down the value of
(i) $\mathrm{g}(1)$,
(ii) $\mathrm{fg}(1)$,
(iii) $\mathrm{g}^{-1}(4)$,
(iv) the positive solution of $\mathrm{f}(x)=\mathrm{g}(x)$.
(e) Draw the tangent to $y=\mathrm{f}(x)$ at $x=3$. Use it to calculate an estimate of the gradient of the curve at this point.

5 (a) Calculate the area of an equilateral triangle with sides 10 cm.
(b) Calculate the radius of a circle with circumference 10 cm .
(c)



Diagram 2


Diagram 3

The diagrams represent the nets of 3 solids. Each straight line is 10 cm long. Each circle has circumference 10 cm . The arc length in Diagram 3 is 10 cm .
(i) Name the solid whose net is Diagram 1. Calculate its surface area.
(ii) Name the solid whose net is Diagram 2. Calculate its volume.
(iii) Name the solid whose net is Diagram 3. Calculate its perpendicular height.
$19 \mathrm{f}(x)=\frac{x+1}{2}$ and $\mathrm{g}(x)=2 x+1$.
(a) Find the value of $\operatorname{gf}(9)$.

## Answer(a)

(b) Find $\operatorname{gf}(x)$, giving your answer in its simplest form.
Answer(b)
(c) Solve the equation $\mathrm{g}^{-1}(x)=1$.


20 (a) Factorise completely $12 x^{2}-3 y^{2}$.

Answer(a)
(b) (i) Expand $(x-3)^{2}$.

> Answer(b)(i)
(ii) $x^{2}-6 x+10$ is to be written in the form $(x-p)^{2}+q$.

Find the values of $p$ and $q$.

2 Answer all of this question on a sheet of graph paper.
(a) $\mathrm{f}(x)=x^{2}-x-3$.

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | $p$ | 3 | -1 | -3 | $q$ | -1 | 3 | $r$ |

(i) Find the values of $p, q$ and $r$.
(ii) Draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant 4$.

Use a scale of 1 cm to represent 1 unit on each axis.
(iii) By drawing a suitable line, estimate the gradient of the graph at the point where $x=-1$.
(b) $\mathrm{g}(x)=6-\frac{x^{3}}{3}$.

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~g}(x)$ | 8.67 | $u$ | $v$ | 5.67 | 3.33 | -3 |

(i) Find the values of $u$ and $v$.
(ii) On the same grid as part (a) (ii) draw the graph of $y=\mathrm{g}(x)$ for $-2 \leqslant x \leqslant 3$.
(c) (i) Show that the equation $\mathrm{f}(x)=\mathrm{g}(x)$ simplifies to $x^{3}+3 x^{2}-3 x-27=0$.
(ii) Use your graph to write down a solution of the equation $x^{3}+3 x^{2}-3 x-27=0$.

## 4 Answer the whole of this question on a sheet of graph paper.

The table gives values of $\quad \mathrm{f}(x)=2^{x}$, for $-2 \leqslant x \leqslant 4$.

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | $p$ | 0.5 | $q$ | 2 | 4 | $r$ | 16 |

(a) Find the values of $p, q$ and $r$.
(b) Using a scale of 2 cm to 1 unit on the $x$-axis and 1 cm to 1 unit on the $y$-axis, draw the graph of $y=\mathrm{f}(x)$ for $-2 \leqslant x \leqslant 4$.
(c) Use your graph to solve the equation $2^{x}=7$.
(d) What value does $\mathrm{f}(x)$ approach as $x$ decreases?
(e) By drawing a tangent, estimate the gradient of the graph of $y=\mathrm{f}(x)$ when $x=1.5$.
(f) On the same grid draw the graph of $y=2 x+1$ for $0 \leqslant x \leqslant 4$.
(g) Use your graph to find the non-integer solution of $2^{x}=2 x+1$.

$O A B C D E$ is a regular hexagon.
With $O$ as origin the position vector of $C$ is $\mathbf{c}$ and the position vector of $D$ is $\mathbf{d}$.
(a) Find, in terms of $\mathbf{c}$ and $\mathbf{d}$,
(i) $\overrightarrow{D C}$,
(ii) $\overrightarrow{O E}$,
(iii) the position vector of $B$.
(b) The sides of the hexagon are each of length 8 cm .

Calculate
(i) the size of angle $A B C$,
(ii) the area of triangle $A B C$,
(iii) the length of the straight line $A C$,
(iv) the area of the hexagon.

16 The function $\mathrm{f}(x)$ is given by

$$
\mathrm{f}(x)=3 x-1 .
$$

Find, in its simplest form,
(a) $\mathrm{f}^{-1} \mathrm{f}(x)$,
Answer(a)
(b) $\mathrm{ff}(x)$.

17 (a) $\sqrt{32}=2^{p}$. Find the value of $p$.
(b) $\sqrt[3]{\frac{1}{8}}=2^{q}$. Find the value of $q$.

$$
\text { Answer(a) } y=
$$

(b) Write down the gradient of the line.
Answer(b)
(c) Write down the co-ordinates of the point where the line crosses the $y$ axis.

$$
\text { Answer }(c) \quad(\quad . . . . ., ~ . . . . . . . ~) \quad[1]
$$



The diagram shows the accurate graph of $y=\mathrm{f}(x)$.
(a) Use the graph to find
(i) $f(0)$,
(ii) $\mathrm{f}(8)$.

(b) Use the graph to solve
(i) $\mathrm{f}(x)=0$,
(ii) $\mathrm{f}(x)=5$.
(c) $k$ is an integer for which the equation $\mathrm{f}(x)=k$ has exactly two solutions.

Use the graph to find the two values of $k$.
(d) Write down the range of values of $x$ for which the graph of $y=\mathrm{f}(x)$ has a negative gradient.
(e) The equation $\mathrm{f}(x)+x-1=0$ can be solved by drawing a line on the grid.
(i) Write down the equation of this line.
(ii) How many solutions are there for $\mathrm{f}(x)+x-1=0$ ?

8 Answer the whole of this question on a sheet of graph paper.
Use one side for your working and one side for your graphs.

Alaric invests \$100 at 4\% per year compound interest.
(a) How many dollars will Alaric have after 2 years?
(b) After $x$ years, Alaric will have $y$ dollars.

He knows a formula to calculate $y$.
The formula is $y=100 \times 1.04^{x}$

| $x$ (Years) | 0 | 10 | 20 | 30 | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ (Dollars) | 100 | $p$ | 219 | $q$ | 480 |

Use this formula to calculate the values of $p$ and $q$ in the table.
(c) Using a scale of 2 cm to represent 5 years on the $x$-axis and 2 cm to represent $\$ 50$ on the $y$-axis, draw an $x$-axis for $0 \leqslant x \leqslant 40$ and a $y$-axis for $0 \leqslant y \leqslant 500$.

Plot the five points in the table and draw a smooth curve through'them.
(d) Use your graph to estimate
(i) how many dollars Alaric will have after 25 years,
(ii) how many years, to the nearest year, it takes for Alaric to have $\$ 200$.
(e) Beatrice invests $\$ 100$ at $7 \%$ per year simple interest.
(i) Show that after 20 years Beatrice has $\$ 240$.
(ii) How many dollars will Beatrice have after 40 years?
(iii) On the same grid, draw a graph to show how the $\$ 100$ which Beatrice invests will increase during the 40 years.
(f) Alaric first has more than Beatrice after $n$ years.

Use your graphs to find the value of $n$.

5 (a) The table shows some values for the equation $y=\frac{x}{2}-\frac{2}{x}$ for $-4 \leqslant x \leqslant-0.5$ and $0.5 \leqslant x \leqslant 4$.

| $x$ | -4 | -3 | -2 | -1.5 | -1 | -0.5 | 0.5 | 1 | 1.5 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -1.5 | -0.83 | 0 | 0.58 |  |  | -3.75 |  | -0.58 | 0 | 0.83 | 1.5 |

(i) Write the missing values of $y$ in the empty spaces.
(ii) On the grid, draw the graph of $y=\frac{x}{2}-\frac{2}{x}$ for $-4 \leqslant x \leqslant-0.5$ and $0.5 \leqslant x \leqslant 4$.

(b) Use your graph to solve the equation $\frac{x}{2}-\frac{2}{x}=1$.

$$
\begin{equation*}
\text { Answer(b) } x=\ldots . . . . . . . . . . \quad \text { or } x= \tag{2}
\end{equation*}
$$

(c) (i) By drawing a tangent, work out the gradient of the graph where $x=2$.
(ii) Write down the gradient of the graph where $x=-2$.

## Answer(c)(ii)

(d) (i) On the grid, draw the line $y=-x$ for $-4 \leqslant x \leqslant 4$.
(ii) Use your graphs to solve the equation


(e) Write down the equation of a straight line which passes through the origin and does intersect the graph of $y=\frac{x}{2}-\frac{2}{x}$.

$$
\mathrm{h}(x)=2^{x}
$$

(a) Find the value of
(i) $\mathrm{f}\left(-\frac{1}{2}\right)$,
Answer(a)(i)
(ii) $\mathrm{g}(-5)$,

Answer(a)(ii)
(iii) $\mathrm{h}(-3)$.

Answer(a)(iii)
(b) Find the inverse function $\mathrm{f}^{-1}(x)$.
(c) $\mathrm{g}(x)=z$.

Find $x$ in terms of $z$.

$$
\text { Answer(c) } x=
$$

(d) Find $\operatorname{gf}(x)$, in its simplest form.
(e) $\mathrm{h}(x)=512$.

Find the value of $x$.

$$
\text { Answer(e) } x=
$$

(f) Solve the equation $2 \mathrm{f}(x)+\mathrm{g}(x)=0$, giving your answers correct to 2 decimal places.

$20 \mathrm{f}(x)=(x-1)^{3} \quad \mathrm{~g}(x)=(x-1)^{2} \quad \mathrm{~h}(x)=3 x+1$
(a) Work out $\mathrm{fg}(-1)$.
(b) Find $\operatorname{gh}(x)$ in its simplest form.
(c) Find $\mathrm{f}^{-1}(x)$.



The diagram shows accurate graphs of $y=\sin x$ and $y=\cos x$ for $0^{\circ} \leqslant x \leqslant 180^{\circ}$

Use the graph to solve the equations
(a) $\sin x-\cos x=0$,
(b) $\sin x-\cos x=0.5$.

Answer(b) $x=$

9 A fence is made from 32 identical pieces of wood, each of length 2 metres correct to the nearest centimetre.

Calculate the lower bound for the total length of the wood used to make this fence.
Write down your full calculator display.

18 (a) $\mathrm{f}(x)=1-2 x$.
(i) Find $f(-5)$.

## Answer(a)(i)

(ii) $\mathrm{g}(x)=3 x-2$.

Find $\operatorname{gf}(x)$. Simplify your answer.
(b) $\mathrm{h}(x)=x^{2}-5 x-11$.

Solve $h(x)=0$.
Show all your working and give your answer correct to 2 decimal places.

19 The braking distance, $d$ metres, for Alex's car travelling at $v \mathrm{~km} / \mathrm{h}$ is given by the formula

$$
200 d=v(v+40) .
$$

(a) Calculate the missing values in the table.

| $v$ <br> $(\mathrm{~km} / \mathrm{h})$ | 0 | 20 | 40 | 60 | 80 | 100 | 120 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d$ <br> $(\mathrm{metres})$ | 0 |  | 16 |  | 48 |  | 96 |

(b) On the grid below, draw the graph of $200 d=v(v+40)$ for $0 \leqslant v \leqslant 120$.

(c) Find the braking distance when the car is travelling at $110 \mathrm{~km} / \mathrm{h}$.

Answer(c)
m
[1]
(d) Find the speed of the car when the braking distance is 80 m .

$$
\mathrm{f}(x)=x^{2}+2
$$

$g(x)=(x+2)^{2}$
$h(x)=3 x-5$
Find
(a) $\operatorname{gf}(-2)$,

> Answer (a)
(b) $\mathrm{h}^{-1}(22)$.

8 (a) $\mathrm{f}(x)=2^{x}$
Complete the table.

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y=\mathrm{f}(x)$ |  | 0.5 | 1 | 2 | 4 |  |  |

(b) $\mathrm{g}(x)=x(4-x)$

Complete the table.

| $x$ | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y=\mathrm{g}(x)$ |  | 0 | 3 |  | 3 | 0 |

(c) On the grid, draw the graphs of
(i) $y=\mathrm{f}(x)$ for $-2 \leqslant x \leqslant 4$,
(ii) $y=\mathrm{g}(x)$ for $-1 \leqslant x \leqslant 4$.

(d) Use your graphs to solve the following equations.
(i) $\mathrm{f}(x)=10$

Answer(d)(i) $x=$
[1]
(ii) $\mathrm{f}(x)=\mathrm{g}(x)$

Answer(d)(ii) $x=$ $\qquad$ or $x=$ [2]
(iii) $\mathrm{f}^{-1}(x)=1.7$

$$
\text { Answer(d)(iii) } x=
$$

6 (a) Complete the table of values for $y=x+\frac{1}{x}$.

| $x$ | -4 | -3 | -2 | -1 | -0.5 | 0.5 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -4.3 | -3.3 |  |  | -2.5 | 2.5 |  |  | 3.3 | 4.3 |



On the grid, draw the graph of $y=x+\frac{1}{x}$ for $-4 \leqslant x \leqslant-0.5$ and $0.5 \leqslant x \leqslant 4$.
Six of the ten points have been plotted for you.
(c) There are three integer values of $k$ for which the equation $\quad x+\frac{1}{x}=k \quad$ has no solutions. Write down these three values of $k$.

Answer(c) $k=$ $\qquad$ or $k=$ $\qquad$ or $k=$
(d) Write down the ranges of $x$ for which the gradient of the graph of $y=x+\frac{1}{x}$ is positive.
Answer(d)
(e) To solve the equation $x+\frac{1}{x}=2 x+1$, a straight line can be drawn on the grid.
(i) Draw this line on the grid for $-2.5 \leqslant x \leqslant 1.5$.
(ii) On the grid, show how you would find the solutions.
(iii) Show how the equation $x+\frac{1}{x}=2 x+1$ can be rearranged intot the form $x^{2}+b x+c=0$ and find the values of $b$ and $c$.

$$
\begin{aligned}
\text { Answer(e)(iii) } b & = \\
c & =
\end{aligned}
$$

$$
\mathrm{f}(x)=x^{3} \quad \mathrm{~g}(x)=2 x-3
$$

(a) Find
(i) $\mathrm{g}(6)$,
(ii) $\mathrm{f}(2 x)$.
(b) Solve $\mathrm{fg}(x)=125$.
(c) Find the inverse function $\mathrm{g}^{-1}(x)$.

Answer (c) $\mathrm{g}^{-1}(x)=$
$19 \mathrm{f}(x)=x^{2} \quad \mathrm{~g}(x)=2^{x} \quad \mathrm{~h}(x)=2 x-3$
(a) Find $g(3)$.
(b) Find $\operatorname{hh}(x)$ in its simplest form.
(c) Find $\operatorname{fg}(x+1)$ in its simplest form.
(b) The table shows some values of the function $y=x^{2}-2$.

| $x$ | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 7 |  | -1 |  | -1 |  | 7 |

(i) Complete the table.
(ii) On the grid, draw the graph of $y=x^{2}-2$ for $-3 \leqslant x \leqslant 3$.
(iii) Use your graph to solve the equation $x^{2}-2=0$.

$$
\begin{equation*}
\operatorname{Answer}(b)(\text { iii }) x= \tag{2}
\end{equation*}
$$

$$
\text { or } x=
$$

(c) Write down the co-ordinates of the points where your graph meets the line $A B$.


7 (a) The table shows some values of the function $y=x^{2}+x-3$.

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 9 | 3 |  | -3 |  | -1 |  | 9 |

(i) Complete the table.
(ii) On the grid, draw the graph of $y=x^{2}+x-3$ for $-4 \leqslant x \leqslant 3$.

(iii) Use your graph to solve the equation $x^{2}+x-3=0$.

$$
\text { Answer(a)(iii) } x=\text {.................... or } x=
$$

(b) (i) Draw the line of symmetry of the graph.
(ii) Write down the equation of the line of symmetry.

> Answer(b)(ii)
(c) Two points, $A$ and $B$, are marked on the grid.
(i) Draw the straight line through the points $A$ and $B$ extending it to the edges of the grid.
(ii) Write down the co-ordinates of the points of intersection of this line with $y=x^{2}+x-3$.

> Answer(c)(ii) (
$\qquad$ ............ ) and (
(iii) Work out the gradient of the straight line through points $A$ and $B$.

(iv) Write down the equation of the straight line through points $A$ and $B$, in the form $y=m x+c$.


5 (a) (i) Complete the table for the function $y=\frac{6}{x}, x \neq 0$.

| $x$ | -6 | -5 | -4 | -3 | -2 | -1 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -1 | -1.2 |  | -2 | -3 | -6 | 6 | 3 |  |  | 1.2 | 1 |

(ii) On the grid, draw the graph of $y=\frac{6}{x}$ for $-6 \leqslant x \leqslant-1$ and $1 \leqslant x \leqslant 6$.

(b) (i) Complete the table for the function $y=\frac{x^{2}}{2}-2$.

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 6 | 2.5 |  |  | -2 |  |  | 2.5 | 6 |

(ii) On the grid opposite, draw the graph of $y=\frac{x^{2}}{2}-2$ for $-4 \leqslant x \leqslant 4$.
(c) Write down the co-ordinates of the point of intersection of the two graphs.

## Answer(c)(

7 (a) Complete the table of values for the equation $y=\frac{4}{x^{2}}, x \neq 0$.

| $x$ | -4 | -3 | -2 | -1 | -0.6 | 0.6 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 0.25 | 0.44 |  |  | 11.11 |  | 4.00 |  | 0.44 |  |

(b) On the grid, draw the graph of $y=\frac{4}{x^{2}}$ for $-4 \leqslant x \leqslant-0.6$ and $0.6 \leqslant x \leqslant 4$.

(c) Use your graph to solve the equation $\frac{4}{x^{2}}=6$.

Answer(c) $x=$
or $x=$
[2]
(d) By drawing a suitable tangent, estimate the gradient of the graph where $x=1.5$.
Answer(d)
(e) (i) The equation $\frac{4}{x^{2}}-x+2=0$ can be solved by finding the intersection of the graph of $y=\frac{4}{x^{2}}$ and a straight line.

Write down the equation of this straight line.
(ii) On the grid, draw the straight line from your answer to part (e)(i).
(iii) Use your graphs to solve the equation $\frac{4}{x^{2}}-x+2=0$.

4 (a) Complete the table of values for the function $y=x^{2}-\frac{3}{x}, x \neq 0$.

| $x$ | -3 | -2 | -1 | -0.5 | -0.25 | 0.25 | 0.5 | 1 | 2 | 3 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 10 | 5.5 |  | 6.3 | 12.1 |  | -11.9 |  |  | 2.5 | 8 |

(b) Draw the graph of $y=x^{2}-\frac{3}{x}$ for $-3 \leqslant x \leqslant-0.25$ and $0.25 \leqslant x \leqslant 3$.

(c) Use your graph to solve $x^{2}-\frac{3}{x}=7$.

$$
\operatorname{Answer}(c) x=\ldots . . . . . . . . . . . . . . \text { or } x=\ldots . . . . . . . . . . . \text { or } x=
$$

(d) Draw the tangent to the curve where $x=-2$.

Use the tangent to calculate an estimate of the gradient of the curve where $x=-2$.

5 (a) Complete the table of values for the function $\mathrm{f}(x)$, where $\mathrm{f}(x)=x^{2}+\frac{1}{x^{2}}, x \neq 0$.

| $x$ | -3 | -2.5 | -2 | -1.5 | -1 | -0.5 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ |  | 6.41 |  | 2.69 |  | 4.25 | 4.25 |  | 2.69 |  | 6.41 |  |

(b) On the grid, draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant-0.5$ and $0.5 \leqslant x \leqslant 3$.

(c) (i) Write down the equation of the line of symmetry of the graph.

> Answer(c)(i)
(ii) Draw the tangent to the graph of $y=\mathrm{f}(x)$ where $x=-1.5$.

Use the tangent to estimate the gradient of the graph of $y=\mathrm{f}(x)$ where $x=-1.5$.

## Answer(c)(ii)

(iii) Use your graph to solve the equation $x^{2}+\frac{1}{x^{2}}=3$.

(iv) Draw a suitable line on the grid and use your graphs to solve the equation $x^{2}+\frac{1}{x^{2}}=2 x$.


Answer(c)(iv) $x=$
or $x=$ $\qquad$

$$
\mathrm{f}(x)=3 x+1 \quad \mathrm{~g}(x)=(x+2)^{2}
$$

(a) Find the values of
(i) $\mathrm{gf}(2)$,

> Answer(a)(i)
(ii) $\mathrm{ff}(0.5)$.
Answer(a)(ii)
(b) Find $\mathrm{f}^{-1}(x)$, the inverse of $\mathrm{f}(x)$.
(c) Find $\mathrm{fg}(x)$.

Give your answer in its simplest form.
(d) Solve the equation $\quad x^{2}+\mathrm{f}(x)=0$.

Show all your working and give your answers correct to 2 decimal places.


20 f: $x \rightarrow 2 x-1$ and g: $x \rightarrow x^{2}-1$.
Find, in their simplest forms,
(a) $\mathrm{f}^{-1}(x)$,

$$
\begin{equation*}
\text { Answer (a) } \mathrm{f}^{-1}(x)= \tag{2}
\end{equation*}
$$

(b) $\operatorname{gf}(x)$.

Answer (b) $\operatorname{gf}(x)=$

5 Answer the whole of this question on a sheet of graph paper.
(a) The table gives values of $\mathrm{f}(x)=\frac{24}{x^{2}}+x^{2}$ for $0.8 \leqslant x \leqslant 6$.

| $x$ | 0.8 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | 38.1 | 25 | 12.9 | 10 | 10.1 | 11.7 | $l$ | $m$ | $n$ | 26 | 31 | 36.7 |

Calculate, correct to 1 decimal place, the values of $l, m$ and $n$.
(b) Using a scale of 2 cm to represent 1 unit on the $x$-axis and 2 cm to represent 5 units on the $y$-axis, draw an $x$-axis for $0 \leqslant x \leqslant 6$ and a $y$-axis for $0 \leqslant y \leqslant 40$.

Draw the graph of $y=\mathrm{f}(x)$ for $0.8 \leqslant x \leqslant 6$.
(c) Draw the tangent to your graph at $x=1.5$ and use it to calculate an estimate of the gradient of the curve at this point.
(d) (i) Draw a straight line joining the points $(0,20)$ and $(6,32)$.
(ii) Write down the equation of this line in the form $y=m x+c$.
(iii) Use your graph to write down the $x$-values of the points of intersection of this line and the curve $y=\mathrm{f}(x)$.
(iv) Draw the tangent to the curve which has the same gradient as your line in part d(i).
(v) Write down the equation for the tangent in part d(iv).

6 (a) On 1 st January 2000, Ashraf was $x$ years old.
Bukki was 5 years older than Ashraf and Claude was twice as old as Ashraf.
(i) Write down in terms of $x$, the ages of Bukki and Claude on 1st January 2000.
(ii) Write down in terms of $x$, the ages of Ashraf, Bukki and Claude on 1st January 2002.
(iii) The product of Claude's age and Ashraf's age on 1st January 2002 is the same as the square of Bukki's age on 1st January 2000.
Write down an equation in $x$ and show that it simplifies to $x^{2}-4 x-21=0$.
(iv) Solve the equation $x^{2}-4 x-21=0$.
(v) How old was Claude on 1st January 2002?
(b) Claude's height, $h$ metres, is one of the solutions of $h^{2}+8 h-17=0$.
(i) Solve the equation $h^{2}+8 h-17=0$.

Show all your working and give your answers correct to 2 decimal places.
(ii) Write down Claude's height, to the nearest centimetre.

4 Answer the whole of this question on a sheet of graph paper.

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(t)$ | 0 | 25 | 37.5 | 43.8 | 46.9 | 48.4 | 49.2 | 49.6 |

(a) Using a scale of 2 cm to represent 1 unit on the horizontal $t$-axis and 2 cm to represent 10 units on the $y$-axis, draw axes for $0 \leqslant t \leqslant 7$ and $0 \leqslant y \leqslant 60$.
Draw the graph of the curve $y=\mathrm{f}(t)$ using the table of values above.
(b) $\mathrm{f}(t)=50\left(1-2^{-t}\right)$.
(i) Calculate the value of $f(8)$ and the value of $f(9)$.
(ii) Estimate the value of $\mathrm{f}(t)$ when $t$ is large.
(c) (i) Draw the tangent to $y=\mathrm{f}(t)$ at $t=2$ and use it to calculate an estimate of the gradient of the curve at this point.
(ii) The function $\mathrm{f}(t)$ represents the speed of a particle at time $t$. Write down what quantity the gradient gives.
(d) (i) On the same grid, draw $y=\mathrm{g}(t)$ where $\mathrm{g}(t)=6 t+10$, for $0 \leqslant t \leqslant 7$.
(ii) Write down the range of values for $t$ where $\mathrm{f}(t)>\mathrm{g}(t)$.
(iii) The function $\mathrm{g}(t)$ represents the speed of a second particle at time $t$.

State whether the first or second particle travels the greater distance for $0 \leqslant t \leqslant 7$.
You must give a reason for your answer.


Adam writes his name on four red cards and Daniel writes his name on six white cards.
(a) One of the ten cards is chosen at random. Find the probability that
(i) the letter on the card is $\mathbf{D}$,
(ii) the card is red,
(iii) the card is red or the letter on the card is $\mathbf{D}$,
(iv) the card is red and the letter on the card is $\mathbf{D}$,
(v) the card is red and the letter on the card is $\mathbf{N}$.

7 A sketch of the graph of the quadratic function $y=p x^{2}+q x+r$ is shown in the diagram.


The graph cuts the $x$-axis at $K$ and $L$.
The point $M$ lies on the graph and on the line of symmetry.
(a) When $p=1, \quad q=-2, \quad r=-3$, find
(i) the $y$-coordinate of the point where $x=4$,
(ii) the coordinates of $K$ and $L$,
(iii) the coordinates of $M$.
(b) Describe how the above sketch of the graph would change ineach of the following cases.
(i) $p$ is negative.
(ii) $p=1, q=r=0$
(c) Another quadratic function is $y=a x^{2}+b x+c$.
(i) Its graph passes through the origin.

Write down the value of $c$.
(ii) The graph also passes through the points $(3,0)$ and $(4,8)$. Find the values of $a$ and $b$.

8 (a) The technical data of a car includes the following information.

| Type of road | Petrol used per 100 km |
| :---: | :---: |
| Main roads | 9.2 litres |
| Other roads | 8.0 litres |

(i) How much petrol is used on a journey of 350 km on a main road?
(ii) On other roads, how far can the car travel on 44 litres of petrol?
(iii) A journey consists of 200 km on a main road and 160 km on other roads.
(a) How much petrol is used?
(b) Work out the amount of petrol used per 100 km of this journey.
(b) A model of a car has a scale of $1: 25$.
(i) The length of the car is 3.95 m .

Calculate the length of the model.
Give your answer in centimetres.
(ii) The painted surface area of the model is $128 \mathrm{~cm}^{2}$.

Calculate the painted surface area of the car, giving your answerin square centimetres.
(iii) The size of the luggage space of the car is 250 litres

Calculate the size of the luggage space of the model, giving your answer in millilitres.

9 (a) $\mathrm{f}(x)=2-3 x$ and $\mathrm{g}(x)$
(i) Solve the equation $\mathrm{f}(x)=7-x$.
(ii) Find $\mathrm{f}^{-1}(x)$.
(iii) Find the value of $\operatorname{gf}(2)-\mathrm{fg}(2)$.
(iv) Find $\operatorname{fg}(x)$.
(b) $\mathrm{h}(x)=x^{x}$.
(i) Find the value of $\mathrm{h}(2)$.
(ii) Find the value of $\mathrm{h}(-3)$, giving your answer as a fraction.
(iii) Find the value of $\mathrm{h}(7.5)$, giving your answer in standard form.
(iv) $\mathrm{h}(-0.5)$ is not a real number. Explain why.
(v) Find the integer value for which $\mathrm{h}(x)=3125$.
(d) On the same grid, draw the graph of $y=2 x-5$ for $-3 \leqslant x \leqslant 3$.
(e) (i) Use your graphs to find solutions of the equation $1-\frac{1}{x^{2}}=2 x-5$.
(ii) Rearrange $1-\frac{1}{x^{2}}=2 x-5$ into the form $a x^{3}+b x^{2}+c=0$, where $a, b$ and $c$ are integers.
(f) (i) Draw a tangent to the graph of $y=\mathrm{f}(x)$ which is parallel to the line $y=2 x-5$.
(ii) Write down the equation of this tangent.

14 The graph drawn below shows the conversion of temperatures in degrees Fahrenheit ( ${ }^{\circ} \mathrm{F}$ ) to temperatures in degrees Celsius $\left({ }^{\circ} \mathrm{C}\right)$.

(a) The temperature of a room is $20^{\circ} \mathrm{C}$. What is the temperature in Eathrenheit?
(b) A liquid has a boiling point of $176^{\circ} \mathrm{F}$. What is the temperature in Celsius?

> Answer(b)
(c) Find $T$ when $T^{\circ} \mathrm{C}=T^{\circ} \mathrm{F}$.

Answer(c) $T=$

15 f: $x \mapsto 5-3 x$.
(a) Find $\mathrm{f}(-1)$.

Answer(a)
(b) Find $\mathrm{f}^{-1}(x)$.

Answer(b)
(c) Find $\mathrm{ff}^{-1}(8)$.

4 Answer the whole of this question on a sheet of graph paper.

$$
\mathrm{f}(x)=3 x-\frac{1}{x^{2}}+3, x \neq 0 .
$$

(a) The table shows some values of $\mathrm{f}(x)$.

| $x$ | -3 | -2.5 | -2 | -1.5 | -1 | -0.5 | -0.4 | -0.3 | 0.3 | 0.4 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | $p$ | -4.7 | -3.3 | -1.9 | -1 | -2.5 | -4.5 | -9.0 | -7.2 | -2.1 | 0.5 | $q$ | 7.1 | 8.8 | 10.3 | $r$ |

Find the values of $p, q$ and $r$.
(b) Draw axes using a scale of 1 cm to represent 0.5 units for $-3 \leqslant x \leqslant 3$ and 1 cm to represent 2 units for $-10 \leqslant y \leqslant 12$.
(c) On your grid, draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant-0.3$ and $0.3 \leqslant x \leqslant 3$.
(d) Use your graph to solve the equations
(i) $3 x-\frac{1}{x^{2}}+3=0$,
(ii) $3 x-\frac{1}{x^{2}}+7=0$.
(e) $\mathrm{g}(x)=3 x+3$.

On the same grid, draw the graph of $y=\mathrm{g}(x)$ for $-3 \leqslant x \leqslant 3$.
(f) (i) Describe briefly what happens to the graphs of $y \Theta_{\mathrm{f}}(x)$ and $y=\mathrm{g}(x)$ for large positive or negative values of $x$.
(ii) Estimate the gradient of $y=\mathrm{f}(x)$ when $x=100$.


The diagram shows a sketch of $y=x^{2}+1$ and $y=4-x$.
(a) Write down the co-ordinates of
(i) the point $C$,
(ii) the points of intersection of $y=4-x$ with each axis.
(b) Write down the gradient of the line $y=4-x$.
(c) Write down the range of values of $x$ for which the gradient of the graph of $y=x^{2}+1$ is negative.
(d) The two graphs intersect at $A$ and $B$.

Show that the $x$ co-ordinates of $A$ and $B$ satisfy the equation $x^{2}+x-3=0$.
(e) Solve the equation $x^{2}+x-3=0$, giving your answers correct to 2 decimal places.
(f) Find the co-ordinates of the mid-point of the straight line $A B$.

3 Answer the whole of this question on a sheet of graph paper.

The table shows some of the values of the function $\mathrm{f}(x)=x^{2}-\frac{1}{x}, \quad x \neq 0$.

| $x$ | -3 | -2 | -1 | -0.5 | -0.2 | 0.2 | 0.5 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 9.3 | 4.5 | 2.0 | 2.3 | $p$ | -5.0 | -1.8 | $q$ | 3.5 | $r$ |

(a) Find the values of $p, q$ and $r$, correct to 1 decimal place.
(b) Using a scale of 2 cm to represent 1 unit on the $x$-axis and 1 cm to represent 1 unit on the $y$-axis, draw an $x$-axis for $-3 \leqslant x \leqslant 3$ and a $y$-axis for $-6 \leqslant y \leqslant 10$.

Draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant-0.2$ and $0.2 \leqslant x \leqslant 3$.
(c) (i) By drawing a suitable straight line, find the three values of $x$ where $\mathrm{f}(x)=-3 x$.
(ii) $x^{2}-\frac{1}{x}=-3 x$ can be written as $x^{3}+a x^{2}+b=0$.

Find the values of $a$ and $b$.
(d) Draw a tangent to the graph of $y=\mathrm{f}(x)$ at the point where $x=-2$.

Use it to estimate the gradient of $y=\mathrm{f}(x)$ when $x=-2$.

$$
\mathrm{f}(x)=4 x+1 \quad \mathrm{~g}(x)=x^{3}+1 \quad \mathrm{~h}(x)=\frac{2 x+1}{3}
$$

(a) Find the value of $g f(0)$.
(b) Find $\mathrm{fg}(x)$. Simplify your answer.
(c) Find $\mathrm{h}^{-1}(x)$.


The graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{g}(x)$ are shown above.
(a) Find the value of
(i) $\mathrm{f}(-2)$,

Answer(a)(i)
(ii) $\mathrm{g}(0)$.
(b) Use the graphs to solve
(i) the equation $\mathrm{f}(x)=20$,
(ii) the equation $\mathrm{f}(x)=\mathrm{g}(x)$,

$$
\text { Answer(b)(ii) } x=
$$

$\qquad$ or $x=$
(iii) the inequality $\mathrm{f}(x)<\mathrm{g}(x)$.
Answer(b)(iii)
(c) Use the points $A$ and $B$ to find the gradient of $y=\mathrm{g}(x)$ as an exact fraction.

## Answer(c)

(d) On the grid, draw the graph of $y=\mathrm{g}(x)-10$.
(e) (i) Draw the tangent to the graph of $y=\mathrm{f}(x)$ at $(-3,-27)$.
(ii) Write down the equation of this tangent.
Answer(e)(ii)
(f) A region, $R$, contains points whose co-ordinates satisfy the inequalities

$$
-3 \leqslant x \leqslant-2, \quad y \leqslant 40 \quad \text { and } \quad y \geqslant \mathrm{~g}(x) .
$$

On the grid, draw suitable lines and label this region $R$.

8
(a) $\mathrm{f}(x)=2 x-1 \quad \mathrm{~g}(x)=x^{2}$

Work out
(i) $\mathrm{f}(2)$,

> Answer(a)(i)
(ii) $\mathrm{g}(-2)$,
Answer(a)(ii)
(iii) $\mathrm{ff}(x)$ in its simplest form,

$$
\text { Answer(a)(iii) } \mathrm{ff}(x)=
$$

(iv) $\mathrm{f}^{-1}(x)$, the inverse of $\mathrm{f}(x)$,
(v) $x$ when $\operatorname{gf}(x)=4$.

(b) $y$ is inversely proportional to $x$ and $y=8$ when $x=2$.

Find,
(i) an equation connecting $y$ and $x$,
Answer(b)(i)
(ii) $y$ when $x=\frac{1}{2}$.

$$
\text { Answer(b)(ii) } y=
$$

$$
\mathrm{f}(x)=6+x^{2}
$$

$$
\mathrm{g}(x)=4 x-1
$$

(a) Find
(i) $\mathrm{g}(3)$,

> Answer(a)(i)
(ii) $\mathrm{f}(-4)$.
Answer(a)(ii)
(b) Find the inverse function $\mathrm{g}^{-1}(x)$.
(c) Find $\operatorname{fg}(x)$ in its simplest form.

$$
\begin{equation*}
\operatorname{Answer}(c) \operatorname{fg}(x)= \tag{3}
\end{equation*}
$$

(d) Solve the equation $\operatorname{gg}(x)=3$.

7 (a) Complete the table for the function $\mathrm{f}(x)=\frac{2}{x}-x^{2}$.

| $x$ | -3 | -2 | -1 | -0.5 | -0.2 |  | 0.2 | 0.5 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | -9.7 | -5 |  |  | -10.0 | 10.0 | 3.75 | 1 |  | -8.3 |

(b) On the grid draw the graph of $y=\mathrm{f}(x)$ for $-3 \leqslant x \leqslant-0.2$ and $0.2 \leqslant x \leqslant 3$.

(c) Use your graph to
(i) solve $\mathrm{f}(x)=2$,

$$
\operatorname{Answer}(c)(\mathrm{i}) x=
$$

(ii) find a value for $k$ so that $\mathrm{f}(x)=k$ has 3 solutions.

$$
\operatorname{Answer}(c)(\mathrm{ii)} k=
$$

(d) Draw a suitable line on the grid and use your graphs to solve the equation $\frac{2}{x}-x^{2}=5 x$.

(e) Draw the tangent to the graph of $y=\mathrm{f}(x)$ at the point where $x=-2$.

Use it to calculate an estimate of the gradient of $y=\mathrm{f}(x)$ when $x=-2$.

Answer(e)

7 (a) Complete the table for the function $\mathrm{f}(x)=\frac{x^{3}}{10}+1$.

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ |  | -1.7 | 0.2 | 0.9 | 1 | 1.1 | 1.8 |  |

(b) On the grid, draw the graph of $y=\mathrm{f}(x)$ for $-4 \leqslant x \leqslant 3$.

(c) Complete the table for the function $\mathrm{g}(x)=\frac{4}{x}, x \neq 0$.

| $x$ | -4 | -3 | -2 | -1 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~g}(x)$ | -1 | -1.3 |  |  |  | 2 | 1.3 |

(d) On the grid, draw the graph of $y=\mathrm{g}(x)$ for $-4 \leqslant x \leqslant-1$ and $1 \leqslant x \leqslant 3$.
(e) (i) Use your graphs to solve the equation $\frac{x^{3}}{10}+1=\frac{4}{x}$.

$$
\text { Answer(e)(i) } x=\text {.............. or } x=\ldots . . . . . . . .
$$

(ii) The equation $\frac{x^{3}}{10}+1=\frac{4}{x}$ can be written as $x^{4}+a x+b=0$.

Find the values of $a$ and $b$.

$$
b=
$$

# EXTENDED MATHEMATICS 2002-2011 <br> <br> CLASSIFIEDS GEOMETRY 

 <br> <br> CLASSIFIEDS GEOMETRY}


$T A$ is a tangent at $A$ to the circle, centre $O$.
Angle $O A B=50^{\circ}$.
Find the value of
(a) $y$,
Answer(a) y
(b) $z$,

Answer $(b) z=$
(c) $t$.

8 Seismic shock waves travel at speed $v$ through rock of density $d$.
$v$ varies inversely as the square root of $d$.
$v=3$ when $d=2.25$.
Find $v$ when $d=2.56$.

16


The co-ordinates of $A, B$ and $C$ are shown on the diagram, which is not to scale.
(a) Find the length of the line $A B$.
(b) Find the equation of the line $A C$.

22

$A, B, C$ and $D$ lie on a circle.
$A C$ and $B D$ intersect at $X$.
(a) Give a reason why angle $B A X$ is equal to angle $C D X$.

## Answer (a)

(b) $A B=4.40 \mathrm{~cm}, C D=9.40 \mathrm{~cm}$ and $B X=3.84 \mathrm{~cm}$.
(i) Calculate the length of $C X$.

(ii) The area of triangle $A B X$ is $5.41 \mathrm{~cm}^{2}$.

Calculate the area of triangle $C D X$.
$\qquad$

3 (a)

$A B C D$ is a quadrilateral with angle $B A D=40^{\circ}$.
$A B$ is extended to $E$ and angle $E B C=30^{\circ}$.
$A B=A D$ and $B D=B C$.
(i) Calculate angle $B C D$.
(ii) Give a reason why $D C$ is not parallel to $A E$.

Answer(a)(ii)

(b) A regular polygon has $n$ sides.

Each exterior angle is $\frac{5 n}{2}$ degrees.
Find the value of $n$.
(c)


1 Javed says that his eyes will blink 415000000 times in 79 years.
(a) Write 415000000 in standard form.

Answer (a)
(b) One year is approximately 526000 minutes.

Calculate, correct to the nearest whole number, the average number of times his eyes will blink per minute.

> Answer (b)

2 Luis and Hans both have their birthdays on January 1st. In 2002 Luis is 13 and Hans is 17 years old.
(a) Which is the next year after 2002 when both their ages will be prime numbers?

## Answer (a) ..........

(b) In which year was Hans twice as old as Luis?

> Answer (b)


Diagram 1


Diagram 2
(a) In Diagram 1, shade the area which represents $A \cup B^{\prime}$.
(b) Describe in set notation the shaded area in Diagram 2.

> Answer (b)


NOT TO
SCALE
$A B C D$ is a parallelogram and $B C E$ is a straight line. Angle $D C E=54^{\circ}$ and angle $D B C=20^{\circ}$.
Find $x$ and $y$.

$$
\begin{aligned}
\text { Answer } x & =\text {................................................... } \\
y & =. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~
\end{aligned}
$$

5 Calculate the length of the straight line joining the points $(-1,4)$ and $(5,-4)$.

> Answer
$\qquad$
15 (a)

(i) Complete quadrilateral $A B C D$ so that the dotted line is the only line of symmetry.
(ii) Write down the special name for quadrilateral $A B C D$.

Answer (a)(ii)
(b)

(i) Complete quadrilateral $E F G H$ so that the dotted line is one of two lines of symmetry.
(ii) Write down the order of rotational symmetry for quadrilateral $E F G H$.

Answer (b)(ii)

17


Two circles have radii $r \mathrm{~cm}$ and $4 r \mathrm{~cm}$.
Find, in terms of $\pi$ and $r$.
(a) the area of the circle with radius $4 r \mathrm{~cm}$,
Answer (a) $\qquad$ $\mathrm{cm}^{2}$
(b) the area of the shaded ring,

Answer (b)
 $\mathrm{cm}^{2}$
(c) the total length of the inner and outer edges of the shaded ring


A sphere, centre $C$, rests on horizontal ground at $A$ and touches a vertical wall at $D$.
A straight plank of wood, $G B W$, touches the sphere at $B$, rests on the ground at $G$ and against the wall at $W$. The wall and the ground meet at $X$.
Angle $W G X=42^{\circ}$.
(a) Find the values of $a, b, c, d$ and $e$ marked on the diagram.
(b) Write down one word which completes the following sentence
'Angle $C G A$ is $21^{\circ}$ because triangle GBC and triangle GAC are. . $\quad$.
(c) The radius of the sphere is 54 cm .
(i) Calculate the distance $G A$. Show all your working.
(ii) Show that $G X=195 \mathrm{~cm}$ correct to the nearest centimetre.
(iii) Calculate the length of the plank $G W$
(iv) Find the distance $B W$.

8 (a) A sector of a circle, radius 6 cm , has an angle of $20^{\circ}$.

Calculate


NOT TO
SCALE
(i) the area of the sector,
(ii) the arc length of the sector.
(b)


A whole cheese is a cylinder, radius 6 cm and height 5 cm .
The diagram shows a slice of this cheese with sector angle $20^{\circ}$.

Calculate
(i) the volume of the slice of cheese,
(ii) the total surface area of the slice of cheese.
(c) The radius, $r$, and height, $h$, of cylindrical cheeses vary but the volume remains constant.
(i) Which one of the following statements $A, B, C$ or $D$ is true?

A: $h$ is proportional to $r$.
$B: \quad h$ is proportional to $r^{2}$.
$C: \quad h$ is inversely proportional to $r$.
$D: \quad h$ is inversely proportional to $r^{2}$.
(ii) What happens to the height $h$ of the cylindrical cheese when the volume remains constant but the radius is doubled?

9

(a) Find the gradient of the line $A B$.

Answer (a)
(b) Calculate the angle that $A B$ makes with the $x$-axis.

Answer (b) $\qquad$

12

$A, B, C, D$ and $E$ lie on a circle, centre $O . \quad A O C$ is a diameter.
Find the value of
(a) $p$,
(b) $q$.



Diagram 1


Diagram 2


Diagram 3


Diagram 4

Diagram 1 shows a triangle with its base divided in the ratio $1: 3$.
Diagram 2 shows a parallelogram with its base divided in the ratio $1: 3$.
Diagram 3 shows a kite with a diagonal divided in the ratio $1: 3$.
Diagram 4 shows two congruent triangles and a trapezium each of height 1 unit.
For each of the four diagrams, write down the percentage of the total area which is shaded. [7]
(b)


Diagram 5


Diagram 6


Diagram 7

Diagram 5 shows a semicircle, centre $O$.
Diagram 6 shows two circles with radii 1 unit and 5 units.
Diagram 7 shows two sectors, centre $O$, with radii 2 units and 3 units.
For each of diagrams 5, 6 and 7, write down the fraction of the total area which is shaded. [6]

The area of triangle $A P Q$ is $99 \mathrm{~cm}^{2}$ and the area of triangle
 $A B C$ is $11 \mathrm{~cm}^{2} . B C$ is parallel to $P Q$ and the length of $P Q$ is 12 cm .

Calculate the length of $B C$.

$A B C D$ is a cyclic quadrilateral.
$A B=9.5 \mathrm{~cm}, B C=11.1 \mathrm{~cm}$, angle $A B C=70^{\circ}$ and angle $C A D=37^{\circ}$.
(a) Calculate the length of $A C$.
(b) Explain why angle $A D C=110^{\circ}$.
(c) Calculate the length of $A D$.
(d) A point $E$ lies on the circle such that triangle $A C E$ is isosceles, with $E A=E C$.
(i) Write down the size of angle $A E C$.
(ii) Calculate the area of triangle $A C E$.

15 The points $A(6,2)$ and $B(8,5)$ lie on a straight line.
(a) Work out the gradient of this line.

> Answer (a)
(b) Work out the equation of the line, giving your answer in the form $y=m x+c$.


NOT TO
SCALE
$A, B$ and $C$ are points on a circle, centre $O$.
Angle $A O B=40^{\circ}$.
(a) (i) Write down the size of angle $A C B$.
(ii) Find the size of angle $O A B$.

(b) The radius of the circle is 5 cm .
(i) Calculate the length of the minor arc $A B$.
(ii) Calculate the area of the minor sector $O A B$.


NOT TO
SCALE
$A D$ is a diameter of the circle $A B C D E$.
Angle $B A C=22^{\circ}$ and angle $A D C=60^{\circ}$. $A B$ and $E D$ are parallel lines.
Find the values of $w, x, y$ and $z$.

Answer $w=$
9
Answer w $\qquad$
$y=$
...
$z=$

22 (a)


In the diagram triangles $A B E$ and $A C D$ are similar.
$B E$ is parallel to $C D$.
$A B=5 \mathrm{~cm}, B C=4 \mathrm{~cm}, B E=4 \mathrm{~cm}, A E=8 \mathrm{~cm}, C D=p \mathrm{~cm}$ and $D E=q \mathrm{~cm}$. Work out the values of $p$ and $q$.

$$
\begin{gathered}
\text { Answer }(a) P=\text {....................................... } \\
\quad q=\text {......................................... } \\
\quad \text { [4] }
\end{gathered}
$$

(b) A spherical balloon of radius 3 metres has a volume of $36 \pi$ cubic metres.

It is further inflated until its radius is 12 m .
Calculate its new volume, leaving your answer in terms of $\pi$.


NOT TO
SCALE

The diagram shows three touching circles.
$A$ is the centre of a circle of radius $x$ centimetres.
$B$ and $C$ are the centres of circles of radius 3.8 centimetres. Angle $A B C=70^{\circ}$.
Find the value of $x$.

$P, Q, R$ and $S$ lie on a circle, centre $O$. $T P$ and $T Q$ are tangents to the circle.
$P R$ is a diameter and angle $P S Q=64^{\circ}$.
(a) Work out the values of $w$ and $x$.

(b) Showing all your working, find the value of $y$.


The largest possible circle is drawn inside a semicircle, as shown in the diagram.
The distance $A B$ is 12 centimetres.
(a) Find the shaded area.
(b) Find the perimeter of the shaded area.
cm [2]
(a)


NOT TO
SCALE
$A, B, C$ and $D$ lie on a circle.
$A C$ and $B D$ intersect at $X$.
Angle $A B X=55^{\circ}$ and angle $A X B=92^{\circ}$.
$B X=26.8 \mathrm{~cm}, A X=40.3 \mathrm{~cm}$ and $X C=20.1 \mathrm{~cm}$.
(i) Calculate the area of triangle $A X B$.

You must show your working.
(ii) Calculate the length of $A B$.

You must show your working.
(iii) Write down the size of angle $A C D$. Give a reason for your answer.
(iv) Find the size of angle $B D C$.
(v) Write down the geometrical word which completes the statement
"Triangle $A X B$ is - to triangle $D X C$."
(vi) Calculate the length of $X D$.

You must show your working.


A circle, centre $O$, touches all the sides of the regular octagon $A B C D E F G H$ shaded in the diagram.
The sides of the octagon are of length 12 cm .
$B A$ and $G H$ are extended to meet at $P . H G$ and $E F$ are extended to meet at $Q$.
(a) (i) Show that angle $B A H$ is $135^{\circ}$.
(ii) Show that angle $A P H$ is $90^{\circ}$.
(b) Calculate
(i) the length of PH ,
(ii) the length of $P Q$,
(iii) the area of triangle $A P H$,
(iv) the area of the octagon.
(c) Calculate
(i) the radius of the circle,
(ii) the area of the circle as a percentage of the area of the octagon.

7 (a)

(i) Write down the geometrical word which completes the following statement.

$$
\begin{equation*}
\text { " } A B C D \text { is a } \square \text { quadrilateral." } \tag{1}
\end{equation*}
$$

(ii) Find the values of $x, y$ and $z$.
(iii) Write down the value of angle $O C T$.
(iv) Find the value of the reflex angle $A O C$.
(b)

NOT TO
SCALE

$P, Q, R$ and $S$ lie on a circle.
$P Q=7 \mathrm{~cm}$ and $S R=10 \mathrm{~cm}$.
$P R$ and $Q S$ intersect at $X$.
The area of triangle $S R X=20 \mathrm{~cm}^{2}$.
(i) Write down the geometrical word which completes the following statement.
"Triangle $P Q X$ is $\quad$ to triangle $S R X$."
(ii) Calculate the area of triangle $P Q X$.
(iii) Calculate the length of the perpendicular height from $X$ to $R S$.


NOT TO
SCALE
$A B$ is the diameter of a circle, centre $O . C, D$ and $E$ lie on the circle.
$E C$ is parallel to $A B$ and perpendicular to $O D$. Angle $D O C$ is $38^{\circ}$.
Work out
(a) angle $B O C$,

$$
\begin{equation*}
\text { Answer (a) Angle } B O C= \tag{1}
\end{equation*}
$$

(b) angle $C B O$,
(c) angle $E D O$.

(a) The line $y=4$ meets the line $2 x+y=8$ at the point $A$.

Find the co-ordinates of $A$.
(b) The line $3 x+y=18$ meets the $x$ axis at the point $B$.

Find the co-ordinates of $B$.

Answer(b) B ( ........ , ....... )
(c) (i) Find the co-ordinates of the mid-point $M$ of the line joining $A$ to $B$.
Answer(c)(i) M ( ........ , ....... )
(ii) Find the equation of the line through $M$ parallel to $3 x+y=18$.
Answer(c)(ii)


The diagram shows the junction of four paths.
In the junction there is a circular area covered in grass.
This circle has centre $O$ and radius 8 m .
(a) Calculate the area of grass.

$$
\begin{equation*}
\text { Answer }\left(\text { a) ....................................... } \mathrm{m}^{2}\right. \tag{2}
\end{equation*}
$$

(b)

The arc $P Q$ and the other three identical arcs, $R S, T U$ and $V W$ are each part of a circle, centre $O$, radius 12 m .
The angle $P O Q$ is $45^{\circ}$.
The arcs $P Q, R S, T U, V W$ and the circumference of the circle in part(a) are painted white.
Calculate the total length painted white.

9 (a)


The lines $A B$ and $C D E$ are parallel.
$A D$ and $C B$ intersect at $X$.
$A B=9 \mathrm{~cm}, C D=6 \mathrm{~cm}$ and $D X=3 \mathrm{~cm}$.
(i) Complete the following statement.

Triangle $A B X$ is $\qquad$ to triangle $D C X$.
(ii) Calculate the length of $A X$.

$$
\text { Answer(a)(ii) } A X=
$$

cm
(iii) The area of triangle $D C X$ is $6 \mathrm{~cm}^{2}$.

Calculate the area of triangle $A B X$.

Answer(a)(iii) $\mathrm{cm}^{2}$
(iv) Angle $B A X=x^{\circ}$ and angle $A B X=y^{\circ}$.

Find angle $A X B$ and angle $X D E$ in terms of $x$ and/or $y$.

Answer(a)(iv) Angle $A X B=$ $\qquad$

$$
\text { Angle } X D E=
$$

(b)

$P, Q, R$ and $S$ lie on a circle, centre $O$.
Angle $O P S=42^{\circ}$ and angle $P R Q=35^{\circ}$.
Calculate
(i) angle $\operatorname{POS}$,

Answer(b)(i) Angle POS =
(ii) angle $P R S$,

Answer(b)(ii) Angle PRS = $\qquad$
(iii) angle $S P Q$,

$$
\text { Answer(b)(iii) Angle } S P Q=
$$

(iv) angle $P S Q$.

$$
\text { Answer(b)(iv) Angle } P S Q=
$$

(c) The interior angle of a regular polygon is 8 times as large as the exterior angle. Calculate the number of sides of the polygon.


In the quadrilateral $A B C D, A B=3 \mathrm{~cm}, A D=11 \mathrm{~cm}$ and $D C=8 \mathrm{~cm}$.
The diagonal $A C=5 \mathrm{~cm}$ and angle $B A C=90^{\circ}$.
Calculate
(a) the length of $B C$,
(b) angle $A C D$,

Answer(b) Angle $A C D=$
(c) the area of the quadrilateral $A B C D$.

5 (a)


The diagram shows two triangles $A C B$ and $A P Q$.
Angle $P A Q=$ angle $B A C$ and angle $A Q P=$ angle $A B C$.
$A B=4 \mathrm{~cm}, B C=3.6 \mathrm{~cm}$ and $A Q=3 \mathrm{~cm}$.
(i) Complete the following statement.

Triangle $A C B$ is
 to triangle $A P^{\prime} Q$.
(ii) Calculate the length of $P Q$.

$$
\text { Answer(a)(ii) } P Q=
$$

$\qquad$
(iii) The area of triangle $A C B$ is $5.6 \mathrm{~cm}^{2}$.

Calculate the area of triangle $A P Q$.
$\qquad$ $\mathrm{cm}^{2}$
(b)

$R, H, S, T$ and $U$ lie on a circle, centre $O$.
$H T$ is a diameter and $M N$ is a tangent to the circle at $T$.
Angle $R T M=61^{\circ}$.
Find
(i) angle $R T H$,
(ii) angle $R H T$,
(iii) angle $R S T$,

Answer(b)(ii) Angle $R H T=$

Answer(b)(iii) Angle $R S T=$
(iv) angle RUT.

Answer(b)(iv) Angle $R U T=$
(c) $A B C D E F$ is a hexagon.

The interior angle $B$ is $4^{\circ}$ greater than interior angle $A$.
The interior angle $C$ is $4^{\circ}$ greater than interior angle $B$, and so on, with each of the next interior angles $4^{\circ}$ greater than the previous one.
(i) By how many degrees is interior angle $F$ greater than interior angle $A$ ?
Answer(c)(i)
(ii) Calculate interior angle $A$.

$A B$ is parallel to $C D$.
Calculate the value of $x$.


$A, B$ and $C$ are points on a circle, centre $O$.
$T A$ is a tangent to the circle at $A$ and $O B T$ is a straight line.
$A C$ is a diameter and angle $O T A=24^{\circ}$.
Calculate
(a) angle $A O T$,
(b) angle $B O C$,
(c) angle $O C B$.


NOT TO
SCALE

A straight line intersects two parallel lines as shown in the diagram.
Find the value of $x$.

9
$A B$ is parallel to $C D$.
Calculate the value of $x$.

$$
\text { Answer } x=
$$

17 (a)


NOT TO
SCALE

Points $A, B$ and $C$ lie on the circumference of the circle shown above.
When angle $B A C$ is $90^{\circ}$ write down a statement about the line $B C$.

Answer (a)
(b)

$O$ is the centre of a circle and the line $A B C$ is a tangent to the circle at $B$.
$D$ is a point on the circumference and angle $B O D=54^{\circ}$.
Calculate angle $D B C$.


The diagram shows a square of side $k \mathrm{~cm}$.
The circle inside the square touches all four sides of the square.
(a) The shaded area is $A \mathrm{~cm}^{2}$.

Show that

$$
4 A=4 k^{2}-\pi k^{2} .
$$

Answer (a)

(b) Make $k$ the subject of the formula $4 A=4 k^{2}-\pi k^{2}$.

17

$A, B$ and $C$ are points on a circle, centre $O$.
$T A$ is a tangent to the circle at $A$ and $O B T$ is a straight line.
$A C$ is a diameter and angle $O T A=24^{\circ}$.
Calculate
(a) angle $A O T$,
(b) angle $A C B$,


Answer(b) Angle $A C B=$
(c) angle $A B T$.


The diagram shows the straight line which passes through the points $(0,1)$ and $(3,13)$.
Find the equation of the straight line.

15 A cylinder has a height of 12 cm and a volume of $920 \mathrm{~cm}^{3}$.
Calculate the radius of the base of the cylinder.


NOT TO
SCALE

The diagram shows a circle, centre $O$.
$V T$ is a diameter and $A T B$ is a tangent to the circle at $T$. $U, V, W$ and $X$ lie on the circle and angle $V O U=70^{\circ}$.

Calculate the value of
(a) $e$,
(b) $f$,


$$
\text { Answer(b) } f=
$$

(c) $g$,

$$
\text { Answer(c) } g=
$$

(d) $h$.

5


Answer(a)(i)
(ii) Write down the equation of the line $A B$ in the form $y=m x+c$.

$$
\text { Answer(a)(ii) } y=
$$

5 (a) The table below shows how many sides different polygons have.
Complete the table.

| Name of polygon | Number of sides |
| :---: | :---: |
|  | 3 |
| Quadrilateral | 4 |
|  | 5 |
| Hexagon | 6 |
| Heptagon | 7 |
| Nonagon | 9 |

(b) Two sides, $A B$ and $B C$, of a regular nonagon are shown in the diagram below.


$$
\text { Answer(b)(i) } x=
$$

(ii) Find the value of angle $A B C$, the interior angle of a regular nonagon.

6 (a)


The diagram shows a triangle $A B C$ with $B A$ extended to $D$.
$A B=A C$ and angle $C A D=140^{\circ}$.
Find the value of $p$.


Find the value of $x$.
(d)


In triangle $A B C$, angle $A=90^{\circ}$ and angle $B=22^{\circ}$.
Calculate angle $C$.
(e)


In triangle $X Y Z, P$ is a point on $X Y$ and $Q$ is a point on $X Z$.
$P Q$ is parallel to $Y Z$.
(i) Complete the statement.

Triangle $X P Q$ is $\qquad$ to triangle $X Y Z$.
(ii) $P Q=8 \mathrm{~cm}, X Q=10 \mathrm{~cm}$ and $Y Z=10 \mathrm{~cm}$.

Calculate the length of $X Z$.
(ii)

$A B=B C=6 \mathrm{~km}$.
Junior students follow a similar path but they only walk 4 km North from $A$, then 4 km on a bearing $110^{\circ}$ before returning to $A$.

Senior students walk a total of 18.9 km .
Calculate the distance walked by junior students.
(c) The total amount, $\$ 1380$, raised in 2010 was $8 \%$ less than the total amount raised in 2009.

Calculate the total amount raised in 2009.


NOT TO
SCALE

The circle, centre $O$, passes through the points $A, B$ and $C$.
In the triangle $A B C, A B=8 \mathrm{~cm}, B C=9 \mathrm{~cm}$ and $C A=6 \mathrm{~cm}$.
(a) Calculate angle $B A C$ and show that it rounds to $78.6^{\circ}$, correct to 1 decimal place.

Answer(a)

(b) $M$ is the midpoint of $B C$.
(i) Find angle $B O M$.
(b)

$E F G$ is a triangle.
$H J$ is parallel to $F G$.
Angle $F E G=75^{\circ}$.
Angle $E F G=2 x^{\circ}$ and angle $F G E=(x+15)^{\circ}$.
(i) Find the value of $x$.
(ii) Find angle $H J G$.

12


In the hexagon $A B C D E F, B C$ is parallel to $E D$ and $D C$ is parallel to $E F$.
Angle $D E F=109^{\circ}$ and angle $E F A=95^{\circ}$.
Angle $F A B$ is equal to angle $A B C$.
Find the size of
(a) angle $E D C$,
(b) angle $F A B$.

Answer (b) Angle $F A B=$

$P Q R S$ is a cyclic quadrilateral. The diagonals $P R$ and $Q S$ intersect at $X$.
Angle $S P R=21^{\circ}$, angle $P R S=80^{\circ}$ and angle $P X Q=33^{\circ}$.
Calculate
(a) angle $P Q S$,

Answer (a) Angle $P Q S=$
(b) angle $Q P R$,

$$
\text { Answer }(b) \text { Angle } Q P R=
$$

(c) angle $P S Q$.

## Answer (c) Angle PSQ =

15 Solve the simultaneous equations

$$
\begin{gathered}
4 x+5 y=0 \\
8 x-15 y=5
\end{gathered}
$$

$$
\begin{array}{r}
\text { Answer } x= \\
y=
\end{array}
$$

22


In the circle, centre $O$, the chords $K L$ and $P Q$ are each of length 8 cm . $M$ is the mid-point of $K L$ and $R$ is the mid-point of $P Q$. $O M=3 \mathrm{~cm}$.
(a) Calculate the length of $O K$.

$$
\text { Answer (a) } O K=
$$

$\qquad$
(b) $R M$ has a length of 5.5 cm . Calculate angle $R O M$.


The diagram shows a sketch of the net of a solid tetrahedron (triangular prism).
The right-angled triangle $A B C$ is its base.
$A C=8 \mathrm{~cm}, B C=6 \mathrm{~cm}$ and $A B=10 \mathrm{~cm} . F C=C E=5 \mathrm{~cm}$.
(a) (i) Show that $B E=\sqrt{ } 61 \mathrm{~cm}$.
(ii) Write down the length of $D B$.
(iii) Explain why $D A=\sqrt{ } 89 \mathrm{~cm}$.
(b) Calculate the size of angle $D B A$.
(c) Calculate the area of triangle $D B A$.
(e) Calculate the volume of the solid.
[The volume of a tetrahedron is $\frac{1}{3}$ (area of the base) $\times$ perpendicular height.]


NOT TO SCALE
$A B C D$ is a cyclic quadrilateral.
$A D$ is parallel to $B C$. The diagonals $D B$ and $A C$ meet at $X$.
Angle $A C B=62^{\circ}$ and angle $A C D=20^{\circ}$.
Calculate
(a) angle $D B A$,
(b) angle $D A B$,

Answer (a) Angle $D B A=$

Answer (b) Angle $D A B=$
(c) angle $D A C$,

Answer (c) Angle $D A C=$
(d) angle $A X B$,

Answer (d) Angle $A X B=$
(e) angle $C D B$.

Answer (e) Angle $C D B=$


## NOT TO SCALE

The diagram shows an athletics track with six lanes.
The distance around the inside of the inner lane is 400 metres.
The radius of each semicircular section of the inside of the inner lane is 35 metres.
(a) Calculate the total length of the two straight sections at the inside of the inner lane.
(b) Each lane is one metre wide.

Calculate the difference in the distances around the outside of the outer lane and the inside of the inner lane.

10 Quadrilaterals $P$ and $Q$ each have diagonals which

- are unequal,
- intersect at right angles.
$P$ has two lines of symmetry. $Q$ has one line of symmetry.
(a) (i) Sketch quadrilateral $P$.

Write down its geometrical name.
(ii) Sketch quadrilateral $Q$.

Write down its geometrical name.
(b) In quadrilateral $P$, an angle between one diagonal and a side is $x^{\circ}$. Write down, in terms of $x$, the four angles of quadrilateral $P$.
(c) The diagonals of quadrilateral $Q$ have lengths 20 cm and 12 cm . Calculate the area of quadrilateral $Q$.
(d) Quadrilateral $P$ has the same area as quadrilateral $Q$.

The lengths of the diagonals and sides of quadrilateral $P$ are all integer values. Find the length of a side of quadrilateral $P$.


NOT TO
SCALE
$A, B, C$ and $D$ lie on a circle centre $O . A C$ is a diameter of the circle.
$A D, B E$ and $C F$ are parallel lines. Angle $A B E=48^{\circ}$ and angle $A C F=126^{\circ}$.
Find
(a) angle $D A E$,
(b) angle $E B C$,
(c) angle $B A E$.

Answer(b) Angle $E B C=$

17


NOT TO
SCALE
$A B C D E$ is a regular pentagon.
$D E F$ is a straight line.
Calculate
(a) angle $A E F$,
(b) angle $D A E$.

18 Simplify
(a) $\left(\frac{x^{27}}{27}\right)^{\frac{2}{3}}$,

Answer(a)
[2]
(b) $\left(\frac{x^{-2}}{4}\right)^{-\frac{1}{2}}$.

19

(a) Calculate the gradient of the line $l$.

Answer(a)
(b) Write down the equation of the line $l$.

Answer (b) (.................................................... [2]


NOT TO
SCALE
$A, B, C$ and $D$ lie on a circle, centre $O$, radius 8 cm .
$A B$ and $C D$ are tangents to a circle, centre $O$, radius 4 cm .
$A B C D$ is a rectangle.
(a) Calculate the distance $A E$.

## Answer(a) $A E$

....
cm [2]
(b) Calculate the shaded area.

12


NOT TO
SCALE

In the diagram $P T$ and $Q R$ are parallel. $T P$ and $T R$ are tangents to the circle $P Q R S$.
Angle $P T R=$ angle $R P Q=38^{\circ}$.
(a) What is the special name of triangle $T P R$. Give a reason for your answer.

Answer(a) name $\qquad$
reason.
(b) Calculate
(i) angle $P Q R$,

Answer(b)(i) Angle $P Q R=$
(ii) angle $P S R$.

Answer(b)(ii)Angle $P S R=$

13 A statue two metres high has a volume of five cubic metres.
A similar model of the statue has a height of four centimetres.
(a) Calculate the volume of the model statue in cubic centimetres.
$\qquad$ $\mathrm{cm}^{3}$ [2]
(b) Write your answer to part (a) in cubic metres.

16

$A B C D$ is a trapezium.
(a) Find the area of the trapezium in terms of $x$ and simplify your answer.
Answer(a)
(b) Angle $B C D=y^{\circ}$. Calculate the value of $y$.


In the diagram, the line $A C$ has equation $2 x+3 y=17$ and the line $A B$ has equation $4 x-y=6$.
The lines $B C$ and $A B$ intersect at $B(1,-2)$.
The lines $A C$ and $B C$ intersect at $C(4,3)$.
(a) Use algebra to find the coordinates of the point $A$.

Answer(a)
(b) Find the equation of the line $B C$.


The line $l$ passes through the points $(10,0)$ and $(0,8)$ as shown in the diagram.
(a) Find the gradient of the line as a fraction in its simplest form.

## Answer(a)

(b) Write down the equation of the line parallel to $l$ which passes through the origin.

Answer(b)
(c) Find the equation of the line parallel to $l$ which passes through the point $(3,1)$.


The points $A, B, C$ and $D$ lie on a circle centre $O$.
Angle $A O B=90^{\circ}$, angle $C O D=50^{\circ}$ and angle $B C D=123^{\circ}$. The line $D T$ is a tangent to the circle at $D$.

Find
(a) angle $O C D$,
(b) angle $T D C$,
(c) angle $A B C$,

Answer(c) Angle $A B C=$
(d) reflex angle $A O C$.

$A B C D E$ is a pentagon.
A circle, centre $O$, passes through the points $A, C, D$ and $E$.
Angle $E A C=36^{\circ}$, angle $C A B=78^{\circ}$ and $A B$ is parallel to $D C$.
(a) Find the values of $x, y$ and $z$, giving a reason for each.
(b) Explain why $E D$ is not parallel to $A C$.
(c) Find the value of angle $E O C$.
(d) $A B=A C$.

Find the value of angle $A B C$.


The line $y=m x+c$ is parallel to the line $y=2 x+4$. The distance $A B$ is 6 units.

Find the value of $m$ and the value of $c$.

$$
\text { Answer } m=
$$

and $c=$

NOT TO
SCALE


Points $A, B$ and $C$ lie on a circle, centre $O$, with diameter $A B$.
$B D, O C E$ and $A F$ are parallel lines.
Angle $C B D=68^{\circ}$.
Calculate
(a) angle $B O C$,

Answer(a) Angle $B O C=$
(b) angle $A C E$.

$O$ is the centre of the circle.
$D A$ is the tangent to the circle at $A$ and $D B$ is the tangent to the circle at $C$.
$A O B$ is a straight line. Angle $C O B=50^{\circ}$.
Calculate
(a) angle $C B O$,
Answer(a) Angle CBO =
(b) angle $D O C$.

5
$J G R$ is a right-angled triangle, $J R=50 \mathrm{~m}$ and $J G=20 \mathrm{~m}$.
Calculate angle $J R G$.

6 Write 0.00658
(a) in standard form,

> Answer(a)
(b) correct to 2 significant figures.


NOT TO
SCALE

The pentagon has three angles which are each $140^{\circ}$.
The other two interior angles are equal.
Calculate the size of one of these angles.

13


NOT TO
SCALE

The diagram shows a circle of radius 5 cm in a square of side 18 cm .
Calculate the shaded area.


14


Draw, accurately, the locus of all the points outside the triangle which are 3 centimetres away from the triangle.

9

$A P B$ and $A Q C$ are straight lines. $P Q$ is parallel to $B C$.
$A P=8 \mathrm{~cm}, P Q=10 \mathrm{~cm}$ and $B C=12 \mathrm{~cm}$.
Calculate the length of $A B$.


The points $A, B, C$ and $D$ lie on the circumference of the circle, centre $O$.

Angle $A B D=30^{\circ}$, angle $C A D=50^{\circ}$ and angle $B O C=86^{\circ}$
(a) Give the reason why angle $D B C=50^{\circ}$.

Answer(a)

(b) Find
(i) angle $A D C$,

Answer(b)(ii) Angle $B D C=$
(iii) angle $O B D$.

6 (a)


The diagram shows a toy boat.
$A C=16.5 \mathrm{~cm}, A B=19.5 \mathrm{~cm}$ and $P R=11 \mathrm{~cm}$.
Triangles $A B C$ and $P Q R$ are similar.
(i) Calculate $P Q$.
(ii) Calculate $B C$.

$$
\text { Answer(a)(ii) } B C=
$$

cm [3]
(iii) Calculate angle $A B C$.
(iv) The toy boat is mathematically similar to a real boat.

The length of the real boat is 32 times the length of the toy boat.
The fuel tank in the toy boat holds 0.02 litres of diesel.
Calculate how many litres of diesel the fuel tank of the real boat holds.
(b)


The diagram shows a field $D E F G$, in the shape of a quadrilateral, with a footpath along the diagonal $D F$.
$D F=105 \mathrm{~m}$ and $F G=67 \mathrm{~m}$.
Angle $E D F=70^{\circ}$, angle $E F D=32^{\circ}$ and angle $D F G=143^{\circ}$
(i) Calculate $D G$.

$$
\text { Answer(b)(i) } D G=\text {................................... } \mathrm{m}
$$

(ii) Calculate $E F$.

7 (a)


NOT TO
SCALE
$A, B, C$ and $D$ are points on the circumference of a circle centre $O$. $A C$ is a diameter.
$B D=B C$ and angle $D B C=62^{\circ}$.
Work out the values of $w, x, y$ and $z$.
Give a reason for each of your answers.



The diagram shows five straight roads.
$P Q=4.5 \mathrm{~km}, Q R=4 \mathrm{~km}$ and $P R=7 \mathrm{~km}$.
Angle $R P S=40^{\circ}$ and angle $P S R=85^{\circ}$.
(a) Calculate angle $P Q R$ and show that it rounds to $110.7^{\circ}$. Answer(a)
(b) Calculate the length of the road $R S$ and show that it rounds to 4.52 km .

Answer (b)

(c) Calculate the area of the quadrilateral $P Q R S$.
[Use the value of $110.7^{\circ}$ for angle $P Q R$ and the value of 4.52 km for $R S$.]

9
NOT TO SCALE


In the pentagon the two angles labelled $t^{\circ}$ are equal.
Calculate the value of $t$.

## EXTENDED MATHEMATICS 2002-2011 CLASSIFEDS MENSURATIONS

Compiled \& Edited

Muhammad Maaz Rashid


The sphere of radius $r$ fits exactly inside the cylinder of radius $r$ and height $2 r$.
Calculate the percentage of the cylinder occupied by the sphere.
[The volume, $V$, of a sphere with radius $r$ is $V=\frac{4}{3} \pi r^{3}$.]


15

$$
a p=p x+c
$$

Write $p$ in terms of $a, c$ and $x$.


NOT TO
SCALE

The diagram shows a plastic cup in the shape of a cone with the end removed.
The vertical height of the cone in the diagram is 20 cm .
The height of the cup is 8 cm .
The base of the cup has radius 2.7 cm .
(a) (i) Show that the radius, $r$, of the circular top of the cup is 4.5 cm .

Answer(a)(i)

(ii) Calculate the volume of water in the cup when it is full.
[The volume, $V$, of a cone with radius $r$ and height $h$ is $V=\frac{1}{3} \pi r^{2} h$.]
(b) (i) Show that the slant height, $s$, of the cup is 8.2 cm .

Answer(b)(i)
(ii) Calculate the curved surface area of the outside of the cup.
[The curved surface area, $A$, of a cone with radius $r$ and slant height $l$ is $A=\pi r l$.]

(b) John wants to estimate the value of $\pi$.

He measures the circumference of a circular pizza as 105 cm and its diameter as 34 cm , both correct to the nearest centimetre.

Calculate the lower bound of his estimate of the value of $\pi$.
Give your answer correct to 3 decimal places.

## Answer(b)

(c) The volume of a cylindrical can is $550 \mathrm{~cm}^{3}$, correct to the nearest $10 \mathrm{~cm}^{3}$. The height of the can is 12 cm correct to the nearest centimetre.

Calculate the upper bound of the radius of the can. Give your answer correct to 3 decimal places.
(c)


The $1080 \mathrm{~cm}^{3}$ of dough is then rolled out to form a cuboid $20 \mathrm{~cm} \times 30 \mathrm{~cm} \times 1.8 \mathrm{~cm}$.
Boris cuts out circular biscuits of diameter 5 cm .
(i) How many whole biscuits can he cut from this cuboid?

> Answer(c)(i)
(ii) Calculate the volume of dough left over.


NOT TO
SCALE

A solid cone has diameter 9 cm , slant height 10 cm and vertical height $h \mathrm{~cm}$.
(a) (i) Calculate the curved surface area of the cone.
[The curved surface area, $A$, of a cone, radius $r$ and slant height $l$ is $A=\pi r l$.]
(ii) Calculate the value of $h$, the vertical height of the cone.
(b)


Sasha cuts off the top of the cone, making a smaller cone with diameter 3 cm .
This cone is similar to the original cone.
(i) Calculate the vertical height of this small cone.
(ii) Calculate the curved surface area of this small cone.

> Answer(b)(ii)
$\mathrm{cm}^{2}$
(c)


The shaded solid from part (b) is joined to a solid cylinder with diameter 9 cm and height 12 cm .
Calculate the total surface area of the whole solid.



A rectangular tank measures 1.2 m by 0.8 m by 0.5 m .
(a) Water flows from the full tank into a cylinder at a rate of $0.3 \mathrm{~m}^{3} / \mathrm{min}$

Calculate the time it takes for the full tank to empty.
Give your answer in minutes and seconds.
(b) The radius of the cylinder is 0.4 m .

Calculate the depth of water, $d$, when all the water from the rectangular tank is in the cylinder.
(c) The cylinder has a height of 1.2 m and is open at the top.

The inside surface is painted at a cost of $\$ 2.30$ per $\mathrm{m}^{2}$. Calculate the cost of painting the inside surface.

6


The diagram shows a triangular prism of length 12 cm .
The rectangle $A B C D$ is horizontal and the rectangle $D C P Q$ is vertical.
The cross-section is triangle $P B C$ in which angle $B C P=90^{\circ}, B C=4 \mathrm{~cm}$ and $C P=3 \mathrm{~cm}$.
(a) (i) Calculate the length of $A P$.
Answer(a)(i) AP=
(ii) Calculate the angle of elevation of $P$ from $A$.
(b) (i) Calculate angle $P B C$.

Answer(b)(i) Angle PBC=
(ii) $X$ is on $B P$ so that angle $B X C=120^{\circ}$.

Calculate the length of $X C$.


8 (a) A sector of a circle, radius 6 cm , has an angle of $20^{\circ}$.

Calculate


NOT TO
SCALE
(i) the area of the sector,
(ii) the arc length of the sector.
(b)


A whole cheese is a cylinder, radius 6 cm and height 5 cm .
The diagram shows a slice of this cheese with sector angle $20^{\circ}$.

Calculate
(i) the volume of the slice of cheese,
(ii) the total surface area of the slice of cheese.
(c) The radius, $r$, and height, $h$, of cylindrical cheeses vary but the volume remains constant.
(i) Which one of the following statements $A, B, C$ or $D$ is true?

A: $h$ is proportional to $r$.
$B: \quad h$ is proportional to $r^{2}$.
$C: \quad h$ is inversely proportional to $r$.
$D: \quad h$ is inversely proportional to $r^{2}$.
(ii) What happens to the height $h$ of the cylindrical cheese when the volume remains constant but the radius is doubled?


A rectangular-based open box has external dimensions of $2 x \mathrm{~cm},(x+4) \mathrm{cm}$ and $(x+1) \mathrm{cm}$.
(a) (i) Write down the volume of a cuboid with these dimensions.
(ii) Expand and simplify your answer.
(b) The box is made from wood 1 cm thick.
(i) Write down the internal dimensions of the box in terms of $x$.
(ii) Find the volume of the inside of the box and show that the volume of the wood is $8 x^{2}+12 x$ cubic centimetres.
(c) The volume of the wood is $1980 \mathrm{~cm}^{3}$.
(i) Show that $2 x^{2}+3 x-495=0$ and solve this equation.
(ii) Write down the external dimensions of the box.


Diagram 1


Diagram 2


Diagram 3


Diagram 4

Diagram 1 shows a triangle with its base divided in the ratio $1: 3$.
Diagram 2 shows a parallelogram with its base divided in the ratio $1: 3$.
Diagram 3 shows a kite with a diagonal divided in the ratio $1: 3$.
Diagram 4 shows two congruent triangles and a trapezium each of height 1 unit.
For each of the four diagrams, write down the percentage of the total area which is shaded. [7]
(b)


Diagram 5


Diagram 6


Diagram 7

Diagram 5 shows a semicircle, centre $O$.
Diagram 6 shows two circles with radii 1 unit and 5 units.
Diagram 7 shows two sectors, centre $O$, with radii 2 units and 3 units.
For each of diagrams 5, 6 and 7, write down the fraction of the total area which is shaded. [6]


NOT TO
SCALE

The diagram shows a solid made up of a hemisphere and a cone.
The base radius of the cone and the radius of the hemisphere are each 7 cm . The height of the cone is 13 cm .
(a) (i) Calculate the total volume of the solid.
[The volume of a hemisphere of radius $r$ is given by $V=\frac{2}{3} \pi r^{3}$.]
[The volume of a cone of radius $r$ and height $h$ is'given by $V=\frac{1}{3} \pi r^{2} h$.]
(ii) The solid is made of wood and $1 \mathrm{~cm}^{3}$ of this wood has a mass of 0.94 g .

Calculate the mass of the solid, in kilograms, correct to 1 decimal place.
(b) Calculate the curved surface area of the cone.
[The curved surface area of a cone of radius $r$ and sloping edge $l$ is given by $A=\pi r l$.]
(c) The cost of covering all the solid with gold plate is $\$ 411.58$.

Calculate the cost of this gold plate per square centimetre.
[The curved surface area of a hemisphere is given by $A=2 \pi r^{2}$.]

17


The height, $h$ metres, of the water, above a mark on a harbour wall, changes with the tide.
It is given by the equation

$$
h=3 \sin (30 t)^{\circ}
$$

where $t$ is the time in hours after midday.
(a) Calculate the value of $h$ at midday.
(b) Calculate the value of $h$ at 1900 .

## Answer (a)

(c) Explain the meaning of the negative sign in your answer.

## NOT TO SCALE



The diagram shows a pencil of length 18 cm .
It is made from a cylinder and a cone.
The cylinder has diameter 0.7 cm and length 16.5 cm .
The cone has diameter 0.7 cm and length 1.5 cm .
(a) Calculate the volume of the pencil.
[The volume, $V$, of a cone of radius $r$ and height $h$ is given by $V=\frac{1}{3} \pi r^{2} h$.]
(b)


Twelve of these pencils just fit into a rectangular box of length 18 cm , width $w \mathrm{~cm}$ and height $x \mathrm{~cm}$. The pencils are in 2 rows of 6 as shown in the diagram.
(i) Write down the values of $w$ and $x$.
(ii) Calculate the volume of the box.
(iii) Calculate the percentage of the volume of the box occupied by the pencils.
(c) Showing all your working, calculate
(i) the slant height, $l$, of the cone,
(ii) the total surface area of one pencil, giving your answer correct to 3 significant figures.
[The curved surface area, $A$, of a cone of radius $r$ and slant height $l$ is given by $A=\pi r l$.]


Diagram 1 shows a closed box. The box is a prism of length 40 cm .
The cross-section of the box is shown in Diagram 2, with all the right-angles marked.
$A B$ is an arc of a circle, centre $O$, radius 12 cm .
$E D=22 \mathrm{~cm}$ and $D C=18 \mathrm{~cm}$.

## Calculate

(a) the perimeter of the cross-section,
(b) the area of the cross-section,
(c) the volume of the box,
(d) the total surface area of the box.

3 Answer the whole of this question on a sheet of graph paper.
(a) Find the values of $k, m$ and $n$ in each of the following equations, where $a>0$.
(i) $a^{0}=k$,
(ii) $a^{m}=\frac{1}{a}$,
(iii) $a^{n}=\sqrt{a}^{3}$.
(b) The table shows some values of the function $\mathrm{f}(x)=2^{x}$.

| $x$ | -2 | -1 | -0.5 | 0 | 0.5 | 1 | 1.5 | 2 | 3 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{f}(x)$ | $r$ | 0.5 | 0.71 | $s$ | 1.41 | 2 | 2.83 | 4 | $t$ |

(i) Write down the values of $r, s$ and $t$.
(ii) Using a scale of 2 cm to represent 1 unit on each axis, draw an $x$-axis from -2 to 3 and a $y$-axis from 0 to 10 .
(iii) On your grid, draw the graph of $y=\mathrm{f}(x)$ for $-2 \leqslant x \leqslant 3$.
(c) The function g is given by $\mathrm{g}(x)=6-2 x$.
(i) On the same grid as part (b), draw the graph of $y=\mathrm{g}(x)$ for $-2 \leqslant x \leqslant 3$.
(ii) Use your graphs to solve the equation $2^{x}=6-2 x$.
(iii) Write down the value of $x$ for which $2^{x}<6-2 x$ for $x \in\{$ positive integers $\}$.


The diagram shows water in a channel.

This channel has a rectangular cross-section, 1.2 metres by 0.8 metres.
(a) When the depth of water is 0.3 metres, the water flows along the channel at 3 metres $/$ minute.

Calculate the number of cubic metres which flow along the channel in one hour.
(b) When the depth of water in the channel increases to 0.8 metres, the water flows at 15 metres/minute.

Calculate the percentage increase in the number of cubic metres which flow along the channel in one hour.
(c) The water comes from a cylindrical tank.

When 2 cubic metres of water leave the tank, the level of water in the tank goes down by 1.3 millimetres.

Calculate the radius of the tank, in metres, correct to one decimal place.
(d) When the channel is empty, its interior surface is repaired.

This costs $\$ 0.12$ per square metre. The total cost is $\$ 50.40$.
Calculate the length, in metres, of the channel.


NOT TO
SCALE

A solid metal bar is in the shape of a cuboid of length of 250 cm .
The cross-section is a square of side $x \mathrm{~cm}$.
The volume of the cuboid is $4840 \mathrm{~cm}^{3}$.
(a) Show that $x=4.4$.

Answer (a)
(b) The mass of $1 \mathrm{~cm}^{3}$ of the metal is 8.8 grams. Calculate the mass of the whole metal bar in kilograms.


Answer(b) ............................ k
kg [2]
(c) A box, in the shape of a cuboid measures 250 cm by 88 cm by $h \mathrm{~cm}$.

120 of the metal bars fit exactly in the box.
Calculate the value of $h$.

$$
\begin{equation*}
\text { Answer(c) } h= \tag{2}
\end{equation*}
$$

(d) One metal bar, of volume $4840 \mathrm{~cm}^{3}$, is melted down to make 4200 identical small spheres.

All the metal is used.
(i) Calculate the radius of each sphere. Show that your answer rounds to 0.65 cm , correct to 2 decimal places.
[The volume, $V$, of a sphere, radius $r$, is given by $V=\frac{4}{3} \pi r^{3}$.]
Answer(d)(i)
(ii) Calculate the surface area of each sphere, using 0.65 cm for the radius.
[The surface area, $A$, of a sphere, radius $r$, is given by
(iii) Calculate the total surface area of all 4200 spheres as a percentage of the surface area of the metal bar.

7 (a) Calculate the volume of a cylinder of radius 31 centimetres and length 15 metres. Give your answer in cubic metres.

> Answer(a) $\mathrm{m}^{3}$
(b) A tree trunk has a circular cross-section of radius 31 cm and length 15 m .

One cubic metre of the wood has a mass of 800 kg .
Calculate the mass of the tree trunk, giving your answer in tonnes.


The diagram shows a pile of 10 tree trunks.
Each tree trunk has a circular cross-section of radius 31 cm and length 15 m .
A plastic sheet is wrapped around the pile.
$C$ is the centre of one of the circles.
$C E$ and $C D$ are perpendicular to the straight edges, as shown.
(i) Show that angle $E C D=120^{\circ}$.

Answer(c)(i)
(ii) Calculate the length of the arc $D E$, giving your answer in metres.
Answer(c)(ii)
(iii) The edge of the plastic sheet forms the perimeter of the cross-section of the pile.

The perimeter consists of three straight lines and three arcs.
Calculate this perimeter, giving your answer in metres.

> Answer(c)(iii)
(iv) The plastic sheet does not cover the two ends of the pile.

Calculate the area of the plastic sheet.

6 A spherical ball has a radius of 2.4 cm .
(a) Show that the volume of the ball is $57.9 \mathrm{~cm}^{3}$, correct to 3 significant figures.
[The volume $V$ of a sphere of radius $r$ is $V=\frac{4}{3} \pi r^{3}$.]
Answer(a)
(b)


Six spherical balls of radius 2.4 cm fit exactly into a closed box.
The box is a cuboid.
Find
(i) the length, width and height of the box,

Answer(b)(i) ............... cm
cm, ............. cm, cm
(ii) the volume of the box,
Answer(b)(ii)

$$
\mathrm{cm}^{3}
$$

(iii) the volume of the box not occupied by the balls,
Answer(b)(iii)

$$
\mathrm{cm}^{3}
$$

(iv) the surface area of the box.
(c)


The six balls can also fit exactly into a closed cylindrical container, as shown in the diagram.
Find
(i) the volume of the cylindrical container,
(ii) the volume of the cylindrical container not occupie @by the balls,

(iii) the surface area of the cylindrical container.


A solid metal cuboid measures 10 cm by 6 cm by 3 cm .
(a) Show that 16 of these solid metal cuboids will fit exactly into a box which has internal measurements 40 cm by 12 cm by 6 cm .

Answer(a)
(b) Calculate the volume of one metal cuboid.

Answer(b) ................................ $\mathrm{cm}^{3}$
(c) One cubic centimetre of the metal has a mass of 8 grams.

The box has a mass of 600 grams.
Calculate the total mass of the 16 cuboids and the box in
(i) grams,
Answer(c)(i)
(ii) kilograms.

> Answer(c)(ii)
kg
(d) (i) Calculate the surface area of one of the solid metal cuboids.

> Answer(d)(i) $\mathrm{cm}^{2}$
(ii) The surface of each cuboid is painted. The cost of the paint is $\$ 25$ per square metre. Calculate the cost of painting all $\mathbf{1 6}$ cuboids.
Answer(d)(ii) \$
(e) One of the solid metal cuboids is melted down.

Some of the metal is used to make 200 identical solid spheres of radius 0.5 cm .
Calculate the volume of metal from this cuboid which is not used.
[The volume, $V$, of a sphere of radius $r$ is $V=\frac{4}{3} \pi r^{3}$.]

> Answer(e)
$\mathrm{cm}^{3}$
(f) $50 \mathrm{~cm}^{3}$ of metal is used to make 20 identical solid spheres of radius $r$.

Calculate the radius $r$.


NOT TO
SCALE

The diagrams show two mathematically similar containers.
The larger container has a base with diameter 9 cm and a height 20 cm . The smaller container has a base with diameter $d \mathrm{~cm}$ and a height 10 cm .
(a) Find the value of $d$.

$$
\operatorname{Answer}(a) d=
$$

(b) The larger container has a capacity of 1600 ml .

Calculate the capacity of the smaller container.


The diagram shows a pyramid with a square base $A B C D$ of side 6 cm .
The height of the pyramid, $P M$, is 4 cm , where $M$ is the centre of the base.
Calculate the total surface area of the pyramid.

9


The diagram shows the net of a box.
(a) (i) Calculate the total surface area of the box.

$\mathrm{cm}^{2}$ [2]
(ii) Calculate the volume of the box.
(b) A cylinder with diameter 18 cm and length 60 cm just fits inside the box.


NOT TO
SCALE
(i) Calculate the volume of the cylinder.

Answer(b)(i)
$\mathrm{cm}^{3}$ [2]
(ii) Find the volume of space outside the cylinder but inside the box.
(iii) Calculate the curved surface area of the cylinder
$\qquad$ $\mathrm{cm}^{2}$ [2]


The diagram shows part of a trench.
The trench is made by removing soil from the ground.
The cross-section of the trench is a rectangle.
The depth of the trench is 0.8 m and the width is 1.4 m .
(a) Calculate the area of the cross-section.
(b) The length of the trench is 200 m .

Calculate the volume of soil removed.

(c)


## NOT TO <br> SCALE

A pipe is put in the trench.
The pipe is a cylinder of radius 0.25 m and length 200 m .
(i) Calculate the volume of the pipe.
[The volume, $V$, of a cylinder of radius $r$ and length $l$ is $V=\pi r^{2} l$.]
(ii) The trench is then filled with soil.

Find the volume of soil put back into the trench.

Answer(c)(ii)
(iii) The soil which is not used for the trench is spread evenly over a horizontal area of $8000 \mathrm{~m}^{2}$.

Calculate the depth of this soil.
Give your answer in millimetres, correct to 1 decimal place.
$\qquad$


In the diagram, $A B C D E F$ is a prism of length 36 cm .
The cross-section $A B C$ is a right-angled triangle.
$A B=19 \mathrm{~cm}$ and $A C=14 \mathrm{~cm}$.
Calculate
(a) the length $B C$,
(b) the total surface area of the prism,


Answer(b) $\qquad$ $\mathrm{cm}^{2} \quad$ [4]
(c) the volume of the prism,

Answer(c) $\qquad$ $\mathrm{cm}^{3}$
(d) the length $C E$,

$$
\text { Answer(d) } C E=
$$

(e) the angle between the line $C E$ and the base $A B E D$.


The diagram shows a box $A B C D E F G H$ in the shape of a cuboid measuring 2 m by 1.5 m by 1.7 m .
(a) Calculate the length of the diagonal $E C$.
(b) Calculate the angle between $E C$ and the base $E F G H$.


Answer(b)
(c) (i) A rod has length 2.9 m , correct to 1 decimal place.

What is the upper bound for the length of the rod?

> Answer(c)(i)
(ii) Will the rod fit completely in the box?

Give a reason for your answer.

> Answer(c)(ii)

7 (a)


NOT TO
SCALE

A solid pyramid has a regular hexagon of side 2.5 cm as its base.
Each sloping face is an isosceles triangle with base 2.5 cm and height 9.5 cm .
Calculate the total surface area of the pyramid.
(b)


A sector $O A B$ has an angle of $55^{\circ}$ and a radius of 15 cm .
Calculate the area of the sector and show that it rounds to $108 \mathrm{~cm}^{2}$, correct to 3 significant figures.
Answer (b)
(c)


NOT TO
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The sector radii $O A$ and $O B$ in part (b) are joined to form a cone.
(i) Calculate the base radius of the cone.
[The curved surface area, $A$, of a cone with radius $r$ and slant height $l$ is $A=\pi r l$.]

Answer(c)(i)
cm [2]
(ii) Calculate the perpendicular height of the cone.
(d)


A solid cone has the same dimensions as the cone in part (c).
A small cone with slant height 7.5 cm is removed by cutting parallel to the base.
Calculate the volume of the remaining solid.
[The volume, $V$, of a cone with radius $r$ and height $h$ is $V=\frac{1}{3} \pi r^{2} h$.]
$\qquad$


Sarah investigates cylindrical plant pots.
The standard pot has base radius $r \mathrm{~cm}$ and height $h \mathrm{~cm}$.
Pot $A$ has radius $3 r$ and height $h$. Pot $B$ has radius $r$ and height $3 h$. Pot $C$ has radius $3 r$ and height $3 h$.
(a) (i) Write down the volumes of pots $A, B$ and C in terms of $\pi, r$ and $h$.
(ii) Find in its lowest terms the ratio of the volumes of $A: B: C$.
(iii) Which one of the pots $A, B$ or $C$ is mathematically similar to the standard pot? Explain your answer.
(iv) The surface area of the standard pot is $S \mathrm{~cm}^{2}$. Write down in terms of $S$ the surface area of the similar pot.
(b) Sarah buys a cylindrical plant pot with radius 15 cm and height 20 cm . She wants to paint its outside surface (base and curved surface area).
(i) Calculate the area she wants to paint.
(ii) Sarah buys a tin of paint which will cover $30 \mathrm{~m}^{2}$

How many plant pots of this size could be painted on their outside surfaces completely using this tin of paint?

9 (a) Write down the 10 th term and the $n$th term of the following sequences.
(i) $1,2,3,4,5 \ldots, \ldots$,
(ii) $7,8,9,10,11 \ldots, \ldots$,
(iii) $8,10,12,14,16 \ldots, \ldots$.
(b) Consider the sequence
$1(8-7), 2(10-8), 3(12-9), 4(14-10)$,
(i) Write down the next term and the 10th term of this sequence in the form $a(b-c)$ where $a, b$ and $c$ are integers.
(ii) Write down the $n$th term in the form $a(b-c)$ and then simplify your answer.


NOT TO
SCALE

The two cones are similar.
(a) Write down the value of $l$.

(b) When full, the larger cone contains $172 \mathrm{~cm}^{3}$ of water.

How much water does the smaller cone contain when it is full?


6 (a) Calculate the volume of a cylinder with radius 30 cm and height 50 cm .
(b)

NOT TO
SCALE

,

A cylindrical tank, radius 30 cm and length 50 cm , lies on its side.
It is partially filled with water.
The shaded segment $A X B Y$ in the diagram shows the cross-section of the water.
The greatest depth, $X Y$, is 12 cm .
$O A=O B=30 \mathrm{~cm}$.
(i) Write down the length of $O X$.
(ii) Calculate the angle $A O B$ correct to two decimal places, showing all your working.
(c) Using angle $A O B=106.3^{\circ}$, find
(i) the area of the sector $A O B Y$,
(ii) the area of triangle $A O B$,
(iii) the area of the shaded segment $A X B Y$.
(d) Calculate the volume of water in the cylinder, giving your answer
(i) in cubic centimetres,
(ii) in litres.
(e) How many more litres must be added to make the tank half full?


The diagram shows a pyramid on a rectangular base $A B C D$, with $A B=6 \mathrm{~cm}$ and $A D=5 \mathrm{~cm}$.
The diagonals $A C$ and $B D$ intersect at $F$.
The vertical height $F P=3 \mathrm{~cm}$.
(a) How many planes of symmetry does the pyramid have?
(b) Calculate the volume of the pyramid.
[The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ height.]
(c) The mid-point of $B C$ is $M$.

Calculate the angle between $P M$ and the base.
(d) Calculate the angle between $P B$ and the base.
(e) Calculate the length of $P B$.


The diagram shows a swimming pool of length 35 m and width 24 m . A cross-section of the pool, $A B C D$, is a trapezium with $A D=2.5 \mathrm{~m}$ and $B C=1.1 \mathrm{~m}$.
(a) Calculate
(i) the area of the trapezium $A B C D$,
(ii) the volume of the pool,
(iii) the number of litres of water in the pool, when it is full.
(b) $A B=35.03 \mathrm{~m}$ correct to 2 decimal places.

The sloping rectangular floor of the pool is painted.
It costs $\$ 2.25$ to paint one square metre.
(i) Calculate the cost of painting the floor of the pool.
(ii) Write your answer to part (b)(i) correct to the nearest hundred dollars.
(c) (i) Calculate the volume of a cylinder, radius 12.5 cm and height 14 cm .
(ii) When the pool is emptied, the water flows through a cylindrical pipe of radius 12.5 cm .

The water flows along this pipe at a rate of 14 centimetres per second.
Calculate the time taken to empty the pool.
Give your answer in days and hours, correct to the nearest hour.

3 Workmen dig a trench in level ground.

NOT TO
SCALE

(a) The cross-section of the trench is a trapezium $A B C D$ with parallel sides of length 1.1 m and 1.4 m and a vertical height of 0.7 m .

Calculate the area of the trapezium.
(b) The trench is 500 m long.

Calculate the volume of soil removed.
(c) One cubic metre of soil has a mass of 4.8 tonnes.

Calculate the mass of soil removed, giving your answer in tonnes and in standard form.
(d) Change your answer to part (c) into grams.

(e) The workmen put a cylindrical pipe, radius 0.2 m and length 500 m , along the bottom of the trench, as shown in the diagram.
Calculate the volume of the cylindrical pipe.
(f) The trench is then refilled with soil.

Calculate the volume of soil put back into the trench as a percentage of the original amount of soil removed.

4 [The surface area of a sphere of radius $r$ is $4 \pi r^{2}$ and the volume is $\frac{4}{3} \pi r^{3}$.]
(a) A solid metal sphere has a radius of 3.5 cm .

One cubic centimetre of the metal has a mass of 5.6 grams.
Calculate
(i) the surface area of the sphere,
(ii) the volume of the sphere,
(iii) the mass of the sphere.
(b)


Diagram 1 shows a cylinder with a diameter of 16 cm .
It contains water to a depth of 8 cm .

Two spheres identical to the sphere in part (a) are placed in the water. This is shown in Diagram 2.
Calculate $h$, the new depth of water in the cylinder.
(c) A different metal sphere has a mass of 1 kilogram.

One cubic centimetre of this metal has a mass of 4.8 grams.
Calculate the radius of this sphere.


The diagram above shows the net of a pyramid.

The base $A B C D$ is a rectangle 8 cm by 6 cm .
All the sloping edges of the pyramid are of length 7 cm .
$M$ is the mid-point of $A B$ and $N$ is the mid-point of $B C$.
(a) Calculate the length of
(i) $Q M$,
(ii) $R N$.
(b) Calculate the surface area of the pyramid.
(c)


The net is made into a pyramid, with $P, Q, R$ and $S$ meeting at $P$.
The mid-point of $C D$ is $G$ and the mid-point of $D A$ is $H$.
The diagonals of the rectangle $A B C D$ meet at $X$.
(i) Show that the height, $P X$, of the pyramid is 4.90 cm , correct to 2 decimal places.
(ii) Calculate angle $P N X$.
(iii) Calculate angle $H P N$.
(iv) Calculate the angle between the edge $P A$ and the base $A B C D$.
(v) Write down the vertices of a triangle which is a plane of symmetry of the pyramid.

4
NOT TO
SCALE


Diagram 1


Diagram 2

Diagram 1 shows a solid wooden prism of length 50 cm .
The cross-section of the prism is a regular pentagon $A B C D E$.
The prism is made by removing 5 identical pieces of wood from a solid wooden cylinder.
Diagram 2 shows the cross-section of the cylinder, centre $O$, radius 15 cm .
(a) Find the angle $A O B$.
(i) the area of triangle $A O B$,
(ii) the area of the pentagon $A B C D E$,
(iii) the volume of wood removed from the cylinder.
(c) Calculate the total surface area of the prism.

(a) The sector of a circle, centre $O$, radius 24 cm , has angle $A O B=60^{\circ}$.

Calculate
(i) the length of the arc $A B$,

Answer(a)(i)
cm
(ii) the area of the sector $O A B$.

(b) The points $A$ and $B$ of the sector are joined together to make a hollow cone as shown in the diagram. The $\operatorname{arc} A B$ of the sector becomes the circumference of the base of the cone.


## Calculate

(i) the radius of the base of the cone,

> Answer(b)(i)
cm [2]
(ii) the height of the cone,

## Answer(b)(ii)

cm [2]
(iii) the volume of the cone.
[The volume, $V$, of a cone of radius $r$ and height $h$ is $V=\frac{1}{3} \pi r^{2} h$.]
(c) A different cone, with radius $x$ and height $y$, has a volume $W$.

Find, in terms of $\boldsymbol{W}$, the volume of
(i) a similar cone, with both radius and height 3 times larger,
Answer(c)(i)
(ii) a cone of radius $2 x$ and height $y$.
Answer(c)(ii)


NOT TO SCALE

The diagram represents a pyramid with a square base of side 10 cm .
The diagonals $A C$ and $B D$ meet at $M . P$ is vertically above $M$ and $P B=8 \mathrm{~cm}$.
(a) Calculate the length of $B D$.

Answer (a) $B D=$
cm
(b) Calculate $M P$, the height of the pyramid.


An open water storage tank is in the shape of a cylinder on top of a cone.
The radius of both the cylinder and the cone is 1.5 m .
The height of the cylinder is 4 m and the height of the cone is 2 m .
(a) Calculate the total surface area of the outside of the tank.
[The curved surface area, $A$, of a cone with radius $r$ and slant height $l$ is $A=\pi r l$.]

(b) The tank is completely full of water.
(i) Calculate the volume of water in the tank and show that it rounds to $33 \mathrm{~m}^{3}$, correct to the nearest whole number.
[The volume, $V$, of a cone with radius $r$ and height $h$ is $V=\frac{1}{3} \pi r^{2} h$.]
Answer(b)(i)
(ii)


The cross-section of an irrigation channel is a semi-circle of radius 0.5 m . The $33 \mathrm{~m}^{3}$ of water from the tank completely fills the irrigation channel.

Calculate the length of the channel.

(c) (i) Calculate the number of litres in a full tank of $33 \mathrm{~m}^{3}$.
Answer(c)(i)
(ii) The water drains from the tank at a rate of 1800 litres per minute.

Calculate the time, in minutes and seconds, taken to empty the tank.
$\qquad$

4 (a)


NOT TO
SCALE

The diagram shows a cone of radius 4 cm and height 13 cm .
It is filled with soil to grow small plants.
Each cubic centimetre of soil has a mass of 2.3 g .
(i) Calculate the volume of the soil inside the cone.
[The volume, $V$, of a cone with radius $r$ and height $h$ is $V=\frac{1}{3} \pi r^{2} h$.]
(ii) Calculate the mass of the soil.

> Answer(a)(i)

$\mathrm{cm}^{3}$
Answer(a)(ii)
(iii) Calculate the greatest number of these cones which can be filled completely using 50 kg of soil.
Answer(a)(iii)
(b) A similar cone of height 32.5 cm is used for growing larger plants.

Calculate the volume of soil used to fill this cone.
(c)


Some plants are put into a cylindrical container with height 12 cm and volume $550 \mathrm{~cm}^{3}$.
Calculate the radius of the cylinder.



The diagram shows a solid made up of a hemisphere and a cylinder.
The radius of both the cylinder and the hemisphere is 3 cm .
The length of the cylinder is 12 cm .
(a) (i) Calculate the volume of the solid.
[ The volume, $V$, of a sphere with radius $r$ is $V=\frac{4}{3} \pi r^{3}$.]

(ii) The solid is made of steel and $1 \mathrm{~cm}^{3}$ of steel has a mass of 7.9 g .

Calculate the mass of the solid.
Give your answer in kilograms.
(iii) The solid fits into a box in the shape of a cuboid, 15 cm by 6 cm by 6 cm . Calculate the volume of the box not occupied by the solid.

Answer(a)(iii)
(b) (i) Calculate the total surface area of the solid.

You must show your working.
[ The surface area, $A$, of a sphere with radius $r$ is $A=4 \pi r^{2}$.]

(ii) The surface of the solid is painted.

The cost of the paint is $\$ 0.09$ per millilitre.
One millilitre of paint covers an area of $8 \mathrm{~cm}^{2}$.
Calculate the cost of painting the solid.

# EXTENDED MATHEMATICS 2002-2011 CLASSIFIEDS TRANSFORMATIONS 


(a) Describe fully the single transformation which maps
(i) triangle $A$ onto triangle $B$,

Answer(a)(i)
(ii) triangle $A$ onto triangle $C$,

Answer(a)(ii)
(iii) triangle $A$ onto triangle $D$.

Answer(a)(iii)
(b) Draw the image of
(i) triangle $B$ after a translation of $\binom{-5}{2}$,
(ii) triangle $B$ after a transformation by the matrix $\left(\begin{array}{ll}1 & 0 \\ 0 & 2\end{array}\right)$.
(c) Describe fully the single transformation represented by the matrix $\left(\begin{array}{ll}1 & 0 \\ 0 & 2\end{array}\right)$.

> Answer(c)


Triangles $T$ and $A$ are drawn on the grid above.
(a) Describe fully the single transformation that maps triangle $T$ onto triangle $A$.

Answer (a)
(b) (i) Draw the image of triangle $T$ after a rotation of $90^{\circ}$ anticlockwise about the point $(0,0)$.

Label the image $B$.
(ii) Draw the image of triangle $T$ after a reflection in the line $x+y=0$.

Label the image $C$.
(iii) Draw the image of triangle $T$ after an enlargement with centre $(4,5)$ and scale factor 1.5 .

Label the image $D$.
(c) (i) Triangle $T$ has its vertices at co-ordinates $(2,1),(6,1)$ and $(6,3)$.

Transform triangle $T$ by the matrix $\left(\begin{array}{ll}1 & 0 \\ 1 & 1\end{array}\right)$.
Draw this image on the grid and label it $E$.
(ii) Describe fully the single transformation represented by the matrix $\left(\begin{array}{ll}1 & 0 \\ 1 & 1\end{array}\right)$. Answer(c)(ii) $\qquad$
(d) Write down the matrix that transforms triangle $B$ onto triangle $T$.


(a) Draw the reflection of shape $P$ in the line $y=x$.
(b) Draw the translation of shape $P$ by the vector $\binom{-2}{1}$.
(c) (i) Describe fully the single transformation that maps shape $P$ onto shape $W$.

Answer(c)(i)

(ii) Find the 2 by 2 matrix which represents this transformation.

(d) Describe fully the single transformation represented by the matrix $\left(\begin{array}{ll}1 & 0 \\ 0 & 2\end{array}\right)$.

(a) Describe fully a single transformation which maps both
(i) $A$ onto $C$ and $B$ onto $D$,
(ii) $A$ onto $D$ and $B$ onto $C$,
(iii) $A$ onto $P$ and $B$ onto $Q$.
(b) Describe fully a single transformation which maps triangle $O A B$ onto triangle $J F E$.
(c) The matrix $\mathbf{M}$ is $\left(\begin{array}{rr}0 & -1 \\ -1 & 0\end{array}\right)$.
(i) Describe the transformation which $\mathbf{M}$ represents.
(ii) Write down the co-ordinates of $P$ after transformation by matrix $\mathbf{M}$.
(d) (i) Write down the matrix $\mathbf{R}$ which represents a rotation by $90^{\circ}$ anticlockwise about 0 .
(ii) Write down the letter representing the new position of $F$ after the transformation $\mathbf{R M}(F)$.

17 (a)


Draw the shear of the shaded square with the $x$-axis invariant and the point $(0,2)$ mapping onto the point $(3,2)$.
(b)

(i) Draw the one-way stretch of the shaded square with the $x$-axis invariant and the point $(0,2)$ mapping onto the point $(0,6)$.
(ii) Write down the matrix of this stretch.

$$
\text { Answer (b)(ii) } \quad(
$$


(a) Describe fully the single transformation which maps
(i) shape $A$ onto shape $B$,
(ii) shape $B$ onto shape $C$,
(iii) shape $A$ onto shape D ,
(iv) shape $B$ onto shape $E$,
(v) shape $B$ onto shape $F$,
(vi) shape $A$ onto shape $G$.
(b) A transformation is represented by the matrix $\left(\begin{array}{rr}0 & -1 \\ 1 & 0\end{array}\right)$.

Which shape above is the image of shape $A$ after this transformation?
(c) Find the 2 by 2 matrix representing the transformation which maps
(i) shape $B$ onto shape $D$,
(ii) shape $A$ onto shape $G$.

(a) Describe fully the single transformation which maps
(i) triangle $X$ onto triangle $P$,
(ii) triangle $X$ onto triangle $Q$,
(iii) triangle $X$ onto triangle $R$,
(iv) triangle $X$ onto triangle $S$.
(b) Find the 2 by 2 matrix which represents the transformation that maps
(i) triangle $X$ onto triangle $Q$,
(ii) triangle $X$ onto triangle $S$.
$7 \quad$ Transformation T is translation by the vector $\binom{3}{2}$.
Transformation M is reflection in the line $y=x$.
(a) The point $A$ has co-ordinates $(2,1)$.

Find the co-ordinates of
(i) $\mathrm{T}(A)$,
(ii) $\operatorname{MT}(A)$.
(b) Find the 2 by 2 matrix $\mathbf{M}$, which represents the transformation $M$.
(c) Show that, for any value of $k$, the point $Q(k-2, k-3)$ maps onto a point on the line $y=x$ following the transformation $\mathrm{TM}(Q)$.
(d) Find $\mathbf{M}^{-1}$, the inverse of the matrix $\mathbf{M}$.
(e) $\mathbf{N}$ is the matrix such that $\mathbf{N}+\left(\begin{array}{ll}0 & 3 \\ 1 & 0\end{array}\right)=\left(\begin{array}{ll}0 & 4 \\ 0 & 0\end{array}\right)$.
(i) Write down the matrix $\mathbf{N}$.
(ii) Describe completely the single transformation represented by
(a) Draw and label $x$ and $y$ axes from -6 to 6 , using a scale of 1 cm to 1 unit.
(b) Draw triangle $A B C$ with $A(2,1), B(3,3)$ and $C(5,1)$.
(c) Draw the reflection of triangle $A B C$ in the line $y=x$. Label this $A_{1} B_{1} C_{1}$.
(d) Rotate triangle $\boldsymbol{A}_{1} \boldsymbol{B}_{1} \boldsymbol{C}_{\mathbf{1}}$ about $(0,0)$ through $90^{\circ}$ anti-clockwise. Label this $A_{2} B_{2} C_{2}$.
(e) Describe fully the single transformation which maps triangle $A B C$ onto triangle $A_{2} B_{2} C_{2}$.
(f) A transformation is represented by the matrix

(i) Draw the image of triangle $A B C$ under this transformation. Label this $A_{3} B_{3} C_{3}$.
(ii) Describe fully the single transformation represented by the matrix $\left(\begin{array}{rr}1 & 0 \\ -1 & 1\end{array}\right)$.
(iii) Find the matrix which represents the transformation that maps triangle $A_{3} B_{3} C_{3}$ onto triangle $A B C$.


Write down the letters of all the triangles which are
(a) congruent to the shaded triangle,

Answer(a)
(b) similar, but not congruent, to the shaded triangle.

(a) On the grid, draw the enlargement of the triangle $T$, centre $(0,0)$, scale factor $\frac{1}{2}$.
(b) The matrix $\left(\begin{array}{cc}-1 & 0 \\ 0 & 1\end{array}\right)$ represents a transformation.
(i) Calculate the matrix product $\left(\begin{array}{cc}-1 & 0 \\ 0 & 1\end{array}\right)\left(\begin{array}{lll}8 & 8 & 2 \\ 4 & 8 & 8\end{array}\right)$.
Answer(b)(i)
(ii) On the grid, draw the image of the triangle $T$ under this transformation.
(iii) Describe fully this single transformation.

Answer(b)(iii)
[2]
(c) Describe fully the single transformation which maps
(i) triangle $T$ onto triangle $P$,

Answer(c)(i)

(ii) triangle $T$ onto triangle $Q$.

Answer(c)(ii)

(d) Find the 2 by 2 matrix which represents the transformation in part (c)(ii).


(a) On the grid, draw
(i) the translation of triangle $T$ by the vector $\binom{-7}{3}$,
(ii) the rotation of triangle $T$ about $(0,0)$, through $90^{\circ}$ clockwise.
(b) Describe fully the single transformation that maps
(i) triangle $T$ onto triangle $U$,

Answer(b)(i)
(ii) triangle $T$ onto triangle $V$.

Answer(b)(ii)
(c) Find the 2 by 2 matrix which represents the transformation that maps
(i) triangle $T$ onto triangle $U$,
(ii) triangle $T$ onto triangle $V$,

(iii) triangle $V$ onto triangle $T$.


(a) Draw the reflection of triangle $T$ in the line $y=6$.

Label the image $A$.
(b) Draw the translation of triangle $T$ by the vector $\binom{-4}{6}$.

Label the image $B$.
(c) Describe fully the single transformation which maps triangle $B$ onto triangle $T$.
Answer(c)
(d) (i) Describe fully the single transformation which maps triangle $T$ onto triangle $P$.
Answer(d)(i)
(ii) Complete the following statement.

Area of triangle $P=$ $\qquad$ $\times$ Area of triangle $T$
(e) (i) Describe fully the single transformation which maps triangle $T$ onto triangle $Q$.

Answer(e)(i) $\qquad$
(ii) Find the 2 by 2 matrix which represents the transformation mapping triangle $T$ onto triangle $Q$.



7 (a)

(i) Reflect triangle $T$ in the line $A B$. Label your image $X$.
(ii) Rotate triangle $T$ through $90^{\circ}$ clockwise about the point $P$. Label your image $Y$.
(b)


Describe the single transformation which maps
(i) flag $P$ onto flag $Q$,

Answer(b)(i)
(ii) flag $P$ onto flag $R$.

Answer(b)(ii)


The diagram shows two triangles drawn on a 1 cm square grid.
(a) (i) Describe fully the single transformation which maps triangle $A$ onto triangle $B$.
Answer(a)(i)

(ii) Calculate the area of triangle $A$.
Answer(a)(ii)
$\qquad$ $\mathrm{cm}^{2}$
(iii) Find the perimeter of triangle $A$.

> Answer(a)(iii)
$\qquad$
(b) Reflect triangle $A$ in the $x$-axis.

Label the image $P$.
(c) Rotate triangle $A$ through $90^{\circ}$ clockwise about $(0,0)$.

Label the image $Q$.
(d) Describe fully the single transformation which maps triangle $P$ onto triangle $Q$.

(a) On the grid, draw the images of the following transformations of shape $\boldsymbol{A}$.
(i) Reflection in the $x$-axis
(ii) Translation by the vector $\binom{3}{4}$
(iii) Rotation, centre $(0,0)$, through $180^{\circ}$
(b) Describe fully the single transformation that maps
(i) shape $A$ onto shape $B$,

Answer(b)(i)
(ii) shape $A$ onto shape $C$.

Answer(b)(ii)

8 (a)


Draw the enlargement of triangle $P$ with centre $A$ and scale factor 2 .
(b)

(i) Describe fully the single transformation which maps shape Q onto shape $R$.

Answer(b)(i)

(ii) Find the matrix which represents this transformation.

$$
\operatorname{Answer}(b)(\mathrm{ii}) \quad(\quad)
$$

(c)


Describe fully the single transformation which maps shape $S$ onto shape $T$.
Answer(c)

(a) (i) Draw the reflection of shape $X$ in the $x$-axis. Label the image $Y$.
(ii) Draw the rotation of shape $Y, 90^{\circ}$ clockwise about $(0,0)$. Label the image $Z$.
(iii) Describe fully the single transformation that maps shape $Z$ onto shape $X$.

Answer(a)(iii)
(b) (i) Draw the enlargement of shape $X$, centre $(0,0)$, scale factor $\frac{1}{2}$.
(ii) Find the matrix which represents an enlargement, centre $(0,0)$, scale factor $\frac{1}{2}$.

$$
\operatorname{Answer}(b)(\mathrm{ii)} \quad(\quad)
$$

(c) (i) Draw the shear of shape $\boldsymbol{X}$ with the $x$-axis invariant and shear factor -1 .
(ii) Find the matrix which represents a shear with the $x$-axis invariant and shear factor -1 .

$$
\operatorname{Answer}(c)(\mathrm{ii}) \quad(\quad)
$$



Answer the whole of this question on a sheet of graph paper.
(a) Using a scale of 1 cm to represent 1 unit on each axis, draw an $x$-axis for $-6 \leqslant x \leqslant 10$ and a $y$-axis for $-8 \leqslant y \leqslant 8$.
Copy the word EXAM onto your grid so that it is exactly as it is in the diagram above.
Mark the point $P(6,6)$.
(b) Draw accurately the following transformations.
(i) Reflect the letter $\mathbf{E}$ in the line $x=0$.
(ii) Enlarge the letter $\mathbf{X}$ by scale factor 3 about centre $P(6,6)$.
(iii) Rotate the letter $\mathbf{A} 90^{\circ}$ anticlockwise about the origin.
(iv) Stretch the letter $\mathbf{M}$ vertically with scale factor 2 and $x$-axis invariant.
(c) (i) Mark and label the point $Q$ so that $\overrightarrow{P Q}=\binom{-3}{2}$.
(ii) Calculate $|\overrightarrow{P Q}|$ correct to two decimal places.
(iii) Mark and label the point $S$ so that $\overrightarrow{P S}\binom{-4}{-1}$.
(iv) Mark and label the point $R$ so that $P Q R S$ is a parallelogram.
$7 \quad$ (a)


Use one of the letters $A, B, C, D, E$ or $F$ to answer the following questions.
(i) Which triangle is $T$ mapped onto by a translation? Wrife down the translation vector.
(ii) Which triangle is $T$ mapped onto by a reflection? Write down the equation of the mirror line.
(iii) Which triangle is $T$ mapped onto by a rotation? Write down the coordinates of the centre of rotation.
(iv) Which triangle is $T$ mapped onto by a stretch with the $x$-axis invariant?

Write down the scale factor of the stretch.
(v) $\mathbf{M}=\left(\begin{array}{ll}1 & 4 \\ 0 & 1\end{array}\right)$. Which triangle is $T$ mapped onto by $\mathbf{M}$ ?

Write down the name of this transformation.
(b) $\mathbf{P}=\left(\begin{array}{ll}1 & 3 \\ 5 & 7\end{array}\right), \quad \mathbf{Q}=\left(\begin{array}{ll}-1 & -2\end{array}\right), \quad \mathbf{R}=\left(\begin{array}{lll}1 & 2 & 3\end{array}\right), \quad \mathbf{S}=\left(\begin{array}{r}-1 \\ 2 \\ 3\end{array}\right)$.

Only some of the following matrix operations are possible with matrices $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$ above.
PQ,
QP,
$\mathbf{P}+\mathbf{Q}$,
PR,
RS

Write down and calculate each matrix operation that is possible.

## 7 Answer the whole of this question on a sheet of graph paper.

(a) Draw $x$ and $y$ axes from 0 to 12 using a scale of 1 cm to 1 unit on each axis.
(b) Draw and label triangle $T$ with vertices $(8,6),(6,10)$ and $(10,12)$.
(c) Triangle $T$ is reflected in the line $y=x$.
(i) Draw the image of triangle $T$. Label this image $P$.
(ii) Write down the matrix which represents this reflection.
(d) A transformation is represented by the matrix $\left(\begin{array}{cc}\frac{1}{2} & 0 \\ 0 & \frac{1}{2}\end{array}\right)$
(i) Draw the image of triangle $T$ under this transformation Label this image $Q$.
(ii) Describe fully this single transformation.
(e) Triangle $T$ is stretched with the $y$-axis invariant and a stretch factor of $\frac{1}{2}$.

Draw the image of triangle $T$. Label this image $R$.

$$
\mathrm{f}(x)=2 x-1,
$$


$\mathrm{h}(x)=2^{x}$.
(a) Find the value of $\mathrm{fg}(6)$.
(b) Write, as a single fraction, $\operatorname{gf}(x)$ in terms of $x$.
(c) Find $\mathrm{g}^{-1}(x)$.
(d) Find hh(3).
(e) Find $x$ when $\mathrm{h}(x)=\mathrm{g}\left(-\frac{24}{7}\right)$


The diagram shows triangles $P, Q, R, S, T$ and $U$.
(a) Describe fully the single transformation which maps triangle
(i) $T$ onto $P$,

(ii) $Q$ onto $T$,
(iii) $T$ onto $R$,
(iv) $T$ onto $S$,
(v) $U$ onto $Q$.
(b) Find the 2 by 2 matrix representing the transformation which maps triangle
(i) $T$ onto $R$,
(ii) $U$ onto $Q$.

(a) Describe fully the single transformation which maps
(i) triangle $T$ onto triangle $U$,

> Answer(a)(i)
(ii) triangle $T$ onto triangle $V$,

> Answer(a)(ii)
(iii) triangle $T$ onto triangle $W$,

Answer(a)(iii)
(iv) triangle $U$ onto triangle $X$.

Answer(a)(iv)
(b) Find the matrix representing the transformation which maps
(i) triangle $U$ onto triangle $V$,
(ii) triangle $U$ onto triangle $X$.



The triangle $K L M$ is shown on the grid.
(a) Calculate angle $K M L$.


Answer(a) Angle $K M L=$
(b) On the grid, draw the shear of triangle $K L M$, with a shear factor of 3 and the $x$-axis invariant.

2 (a)

(i) Draw the image when triangle $A$ is reflected in the line $y=0$.

Label the image $B$.
(ii) Draw the image when triangle $A$ is rotated through $20^{\circ}$ anticlockwise about the origin. Label the image $C$.
(iii) Describe fully the single transformation which maps triangle $B$ onto triangle $C$.

> Answer(a)(iii)
(b) Rotation through $90^{\circ}$ anticlockwise about the origin is represented by the matrix $\mathbf{M}=\left(\begin{array}{rr}0 & -1 \\ 1 & 0\end{array}\right)$.
(i) Find $\mathbf{M}^{-1}$, the inverse of matrix $\mathbf{M}$.

$$
\begin{equation*}
\operatorname{Answer}(b)(\mathrm{i}) \mathbf{M}^{-1}=( \tag{2}
\end{equation*}
$$

(ii) Describe fully the single transformation represented by the matrix $\mathbf{M}^{-1}$.

Answer(b)(ii)
(b) The area of triangle $E$ is $k \times$ area of triangle $A$.

Write down the value of $k$.

$$
\operatorname{Answer}(b) k=
$$

(c)

(i) Draw the image of triangle $F$ under the transformation represented by the matrix $\mathbf{M}=\left(\begin{array}{ll}1 & 3 \\ 0 & 1\end{array}\right)$.
(ii) Describe fully this single transformation.

Answer(c)(ii) $\qquad$
$\qquad$
(iii) Find $\mathbf{M}^{-1}$, the inverse of the matrix $\mathbf{M}$.


19

(a) Describe fully the single transformation which maps triangle $A$ onto triangle $B$.

Answer (a)
(b) Find the $2 \times 2$ matrix which represents this transformation.
Answer (b)

## EXTENDED MATHEMATICS 2002-2011 <br> CLASSIFIEDS TRIGOnBEARING

Compiled \& Edited
Muhammad Maaz Rashid



NOT TO
SCALE
$A O C$ is a diameter of the circle, centre $O$.
$A T$ is a straight line that cuts the circle at $B$.
$P T$ is the tangent to the circle at $C$.
Angle $C O B=76^{\circ}$.
(a) Calculate angle $A T C$.
(b) $T$ is due north of $C$.

Calculate the bearing of $B$ from $C$.


The diagram shows 3 ships $A, B$ and $C$ at sea.
$A B=5 \mathrm{~km}, B C=4.5 \mathrm{~km}$ and $A C=2.7 \mathrm{~km}$.
(a) Calculate angle $A C B$.

Show all your working.


Answer(a) Angle $A C B=$
(b) The bearing of $A$ from $C$ is $220^{\circ}$.

Calculate the bearing of $B$ from $C$.


The quadrilateral $A B C D$ represents an area of land.
There is a straight road from $A$ to $C$.
$A B=79 \mathrm{~m}, A D=120 \mathrm{~m}$ and $C D=95 \mathrm{~m}$.
Angle $B C A=26^{\circ}$ and angle $C D A=77^{\circ}$.
(a) Show that the length of the road, $A C$, is 135 m correct to the nearest metre.

Answer(a)

(b) Calculate the size of the obtuse angle $A B C$.
(c) A straight path is to be built from $B$ to the nearest point on the road $A C$.

Calculate the length of this path.
(d) Houses are to be built on the land in triangle $A C D$. Each house needs at least $180 \mathrm{~m}^{2}$ of land.

Calculate the maximum number of houses which can be built. Show all of your working.

> Answer(d)


NOT TO
SCALE

Parvatti has a piece of canvas $A B C D$ in the shape of an irregular quadrilateral. $A B=3 \mathrm{~m}, A C=5 \mathrm{~m}$ and angle $B A C=45^{\circ}$.
(a) (i) Calculate the length of $B C$ and show that it rounds to 3.58 m , congect to 2 decimal places. You must show all your working.

Answer(a)(i)

(ii) Calculate angle $B C A$.
(b) $A C=C D$ and angle $C D A=52^{\circ}$.
(i) Find angle $D C A$.
(ii) Calculate the area of the canvas.

$\mathrm{m}^{2}$
(c) Parvatti uses the canvas to give some shade.

She attaches corners $A$ and $D$ to the top of vertical poles, $A P$ and $D Q$, each of height 2 m . Corners $B$ and $C$ are pegged to the horizontal ground. $A B$ is a straight line and angle $B P A=90^{\circ}$.


Calculate angle $P A B$.


The triangular area $A B C$ is part of Henri's garden.
$A B=9 \mathrm{~m}, B C=6 \mathrm{~m}$ and angle $A B C=95^{\circ}$.
Henri puts a fence along $A C$ and plants vegetables in the triangular area $A B C$.
Calculate
(a) the length of the fence $A C$,
Answer (a) AC=
(b) the area for vegetables.


Antwerp is 78 km due South of Rotterdam and 83 km due East of Bruges, as shown in the diagram.

Calculate
(a) the distance between Bruges and Rotterdam,

(b) the bearing of Rotterdam from Bruges, correct to the nearest degree.

20 A plane flies from Auckland $(A)$ to Gisborne $(G)$ on a bearing of $115^{\circ}$.
The plane then flies on to Wellington $(W)$. Angle $A G W=63^{\circ}$.


NOT TO
SCALE

## Answer (a)


(b) The distance from Wellington to Gisborne is 400 kilometres.

The distance from Auckland to Wellington is 410 kilometres.

Calculate the bearing of Wellington from Auckland.


The diagram shows three straight horizontal roads in a town, connecting points $P, A$ and $B$. $P B=250 \mathrm{~m}$, angle $A P B=23^{\circ}$ and angle $B A P=126^{\circ}$.
(a) Calculate the length of the $\operatorname{road} A B$.
(a) Calculate the length of the road $A B$.

(b) The bearing of $A$ from $P$ is $303^{\circ}$.

Find the bearing of
(i) $B$ from $P$,

> Answer(b)(i)
(ii) $A$ from $B$.

12 The diagram represents the ski lift in Queenstown New Zealand.


NOT TO
SCALE
(a) The length of the cable from the bottom, $B$, to the top, $T$, is 730 metres.

The angle of elevation of $T$ from $B$ is $37.1^{\circ}$.

Calculate the change in altitude, $h$ metres, from the bottom to the top.
(b) The lift travels along the cable at 3.65 metres per second.

Calculate how long it takes to travel from $B$ to $T$.
Give your answer in minutes and seconds.


The diagram shows three points $P, Q$ and $R$ on horizontal ground.
$P Q=50 \mathrm{~m}, P R=100 \mathrm{~m}$ and angle $P Q R=140^{\circ}$.
(a) Calculate angle $P R Q$.

Answer(a) Angle $P R Q=$
(b) The bearing of $R$ from $Q$ is $100^{\circ}$.

Find the bearing of $P$ from $R$.

$A B C D$ is a quadrilateral and $B D$ is a diagonal.
$A B=26 \mathrm{~cm}, B D=24 \mathrm{~cm}$, angle $A B D=40^{\circ}$, angle $C B D=40^{\circ}$ and angle $C D B=30^{\circ}$.
(a) Calculate the area of triangle $A B D$.

Answer(a)

$\mathrm{cm}^{2}$
(b) Calculate the length of $A D$.
(c) Calculate the length of $B C$.
cm
(d) Calculate the shortest distance from the point $C$ to the line $B D$.


The diagram shows some straight line distances between Auckland $(A)$, Hamilton $(H)$, Tauranga ( $T$ ) and Rotorua $(R)$.
$A T=180 \mathrm{~km}, A H=115 \mathrm{~km}$ and $H T=90 \mathrm{~km}$.
(a) Calculate angle $H A T$.

Show that this rounds to $25.0^{\circ}$, correct to 3 significant figutres.
Answer(a)
(b) The bearing of $H$ from $A$ is $150^{\circ}$.

Find the bearing of
(i) $T$ from $A$,
Answer(b)(i)
(ii) $A$ from $T$.
(c) Calculate how far $T$ is east of $A$.
(d) Angle $T H R=30^{\circ}$ and angle $H R T=70^{\circ}$.

Calculate the distance $T R$.

(e) On a map the distance representing $H T$ is 4.5 cm .

The scale of the map is $1: n$.
Calculate the value of $n$.

$$
\text { Answer(e) } n=
$$

1 In the right-angled triangle $A B C, \cos C=\frac{4}{5}$. Find angle $A$.



In the quadrilateral $A B C D, A B=3 \mathrm{~cm}, A D=11 \mathrm{~cm}$ and $D C=8 \mathrm{~cm}$.
The diagonal $A C=5 \mathrm{~cm}$ and angle $B A C=90^{\circ}$.
Calculate
(a) the length of $B C$,
(b) angle $A C D$,

Answer(b) Angle $A C D=$
(c) the area of the quadrilateral $A B C D$.

10


In triangle $A B C, A B=12 \mathrm{~cm}$, angle $C=90^{\circ}$ and angle $A=27^{\circ}$.
Calculate the length of $A C$.

11


In the rectangle $A B C D, A B=9 \mathrm{~cm}$ and $B D=12 \mathrm{~cm}$.
Calculate the length of the side $B C$.

Answer $B C=$
cm [3]

12 (a) Write 16460000 in standard form.
Answer(a)
(b) Calculate $7.85 \div\left(2.366 \times 10^{2}\right)$, giving your answer in standard form.

10 (a)


NOT TO
SCALE
$B$ is 120 m from $A$ on a bearing of $053^{\circ}$.
Calculate
(i) the distance $d$,
(ii) the bearing of $A$ from $B$.

Answer(a)(i) $d=$

Answer(a)(ii)
(b)


A vertical flagpole, $A F$, is 9 m high.
It is held in place by two straight wires $F G$ and $F H$.
$F G=20 \mathrm{~m}$ and $A H=24 \mathrm{~m}$.
$G, A$ and $H$ lie in a straight line on horizontal ground.
Calculate
(i) angle $F H A$,
(ii) the distance $G A$.

8 Manuel rows his boat from $A$ to $B$, a distance of 3 kilometres.
The scale diagram below shows his journey.
1 centimetre represents 0.5 kilometres.

(a) (i) Measure the bearing of $B$ from $A$.
(ii) The journey from $A$ to $B$ takes him 30 minutes.

Calculate his average speed in kilometres per hour.
Answer(a)(ii)
$\qquad$ km/h
(b) From $B$, Manuel rows 3.5 kilometres in a straight line, on a bearing of $145^{\circ}$, to a point $C$.

On the diagram, draw accurately this journey and label the point $C$.
(c) Manuel then rows from $C$ to $A$.
(i) Measure $C A$.

Answer(c)(i) $\qquad$ cm [1]
(ii) Work out the actual distance from $C$ to $A$.

Answer(c)(ii)
km [1]
(iii) By measuring a suitable angle, find the bearing of $A$ from $C$.

Answer(c)(iii)
(d) Two buoys, $P$ and $Q$, are on opposite sides of the line $A B$. Each buoy is 2 km from $A$ and 1.5 km from $B$.
(i) On the diagram, construct and mark the positions of $P$ and $Q$.
(ii) Measure the distance between $P$ and $Q$.
(iii) Find the actual distance, $P Q$, in kilometres.

4 (a)


The diagram shows triangle $F G H$, with $F G=14 \mathrm{~cm}, G H=12 \mathrm{~cm}$ and $F H=6 \mathrm{~cm}$.
(i) Calculate the size of angle $H F G$.
(ii) Calculate the area of triangle $F G H$.
$\qquad$ $\mathrm{cm}^{2}$
(b)


The diagram shows triangle $P Q R$, with $R P=12 \mathrm{~cm}, R Q=18 \mathrm{~cm}$ and angle $R P Q=117^{\circ}$.
Calculate the size of angle $R Q P$.


3 (a)


The scale drawing shows the positions of two towns $A$ and $C$ on a map.
On the map, 1 centimetre represents 20 kilometres.
(i) Find the distance in kilometres from town $A$ to town $C$
(ii) Measure and write down the bearing of town $C$ from town $A$.
Answer(a)(ii)
(iii) Town $B$ is 140 km from town $C$ on a bearing of $150^{\circ}$.

Mark accurately the position of town $B$ on the scale drawing.
(iv) Find the bearing of town $C$ from town $B$.
Answer(a)(iv)
(v) A lake on the map has an area of $0.15 \mathrm{~cm}^{2}$.

Work out the actual area of the lake.

$$
\begin{equation*}
\text { Answer(a)(v) ................................................ } \mathrm{km}^{2} \tag{2}
\end{equation*}
$$

(b) A plane leaves town $C$ at 1157 and flies 1500 km to another town, landing at 1412 . Calculate the average speed of the plane.

## Answer(b)

km/h [3]
(c)


The diagram shows the distances between three towns $P, Q$ and $R$.
Calculate angle $P Q R$.



NOT TO
SCALE

The circle, centre $O$, passes through the points $A, B$ and $C$.
In the triangle $A B C, A B=8 \mathrm{~cm}, B C=9 \mathrm{~cm}$ and $C A=6 \mathrm{~cm}$.
(a) Calculate angle $B A C$ and show that it rounds to $78.6^{\circ}$, correct to 1 decimal place.

Answer(a)

(b) $M$ is the midpoint of $B C$.
(i) Find angle $B O M$.
(ii) Calculate the radius of the circle and show that it rounds to 4.59 cm , correct to 3 significant figures.

Answer(b)(ii)
(c) Calculate the area of the triangle $A B C$ as a percentage of the area of the circle.


Answer(c)
\% [4]

22


In the circle, centre $O$, the chords $K L$ and $P Q$ are each of length 8 cm . $M$ is the mid-point of $K L$ and $R$ is the mid-point of $P Q$. $O M=3 \mathrm{~cm}$.
(a) Calculate the length of $O K$.

$$
\text { Answer (a) } O K=
$$

$\qquad$
(b) $R M$ has a length of 5.5 cm . Calculate angle $R O M$.


Felipe $(F)$ stands 17 metres from a bridge $(B)$ and 32 metres from a tree $(T)$. The points $F, B$ and $T$ are on level ground and angle $B F T=40^{\circ}$.
(a) Calculate
(i) the distance $B T$,
(ii) the angle $B T F$.
(b) The bearing of $B$ from $F$ is $085^{\circ}$. Find the bearing of
(i) $T$ from $F$,
(ii) $F$ from $T$,
(iii) $B$ from $T$.
(c) The top of the tree is 30 metres vertically above $T$.

Calculate the angle of elevation of the top of the tree from ${ }^{\circ} F$.



To avoid an island, a ship travels 40 kilometres from $A$ to $B$ and then 60 kilometres from $B$ to $C$.
The bearing of $B$ from $A$ is $080^{\circ}$ and angle $A B C$ is $115^{\circ}$
(a) The ship leaves $A$ at 1155 .

It travels at an average speed of $35 \mathrm{~km} / \mathrm{h}$.
Calculate, to the nearest minute, the time it arrives at $C$.
(b) Find the bearing of
(i) $A$ from $B$,
(ii) $C$ from $B$.
(c) Calculate the straight line distance $A C$.
(d) Calculate angle $B A C$.
(e) Calculate how far $C$ is east of $A$.

$A, B$ and $C$ are three places in a desert. Tom leaves $A$ at 0640 and takes 30 minutes to walk directly to $B$, a distance of 3 kilometres. He then takes an hour to walk directly from $B$ to $C$, also a distance of 3 kilometres.
(a) At what time did Tom arrive at $C$ ?
Answer (a)
(b) Calculate his average speed for the whole journey.
(c) The bearing of $C$ from $A$ is $085^{\circ}$ Find the bearing of $A$ from $C$.


The diagram shows the positions of London $(L)$, Dubai $(D)$ and Colombo $(C)$.
(a) (i) Show that $L C$ is 8710 km correct to the nearest kilometre.

Answer(a)(i)



The diagram shows five straight roads.
$P Q=4.5 \mathrm{~km}, Q R=4 \mathrm{~km}$ and $P R=7 \mathrm{~km}$.
Angle $R P S=40^{\circ}$ and angle $P S R=85^{\circ}$.
(a) Calculate angle $P Q R$ and show that it rounds to $110.7^{\circ}$. Answer(a)
(b) Calculate the length of the road $R S$ and show that it rounds to 4.52 km .

Answer (b)

(c) Calculate the area of the quadrilateral $P Q R S$.
[Use the value of $110.7^{\circ}$ for angle $P Q R$ and the value of 4.52 km for $R S$.]

21


Theresa swims from $P$ to $Q$, then from $Q$ to $R$ and then finally returns from $R$ to $P$. $P Q=140 \mathrm{~m}, R P=220 \mathrm{~m}$ and angle $P R Q=31^{\circ}$.
(a) Angle $P Q R$ is obtuse.

Calculate its size, to the nearest degree.

(b) The bearing of $Q$ from $P$ is $060^{\circ}$. Calculate the bearing of $R$ from $Q$.
$22 \mathrm{f}: x \mapsto 3-2 x \quad$ and
(a) Find $f\left(-\frac{3}{4}\right)$.

## Answer (b)

Answer (a)
(b) Find the inverse function, $\mathrm{g}^{-1}(x)$.

Answer (b) $\mathrm{g}^{-1}(x)=$
(c) Find the composite function, $f g(x)$, giving your answer as a single fraction.

# EXTENDED MATHEMATICS 2002-2011 <br> CLASSIFIEDS ALGEBRA 

Compiled \& Edited

Muhammad Maaz Rashid

11 Factorise completely.

$$
p^{2} x-4 q^{2} x
$$



16 The time, $t$, for a pendulum to swing varies directly as the square root of its length, $l$. When $l=9, t=6$.
(a) Find a formula for $t$ in terms of $l$.

$$
\text { Answer(a) } t=
$$

(b) Find $t$ when $l=2.25$.

14 (a) Write down the value of $x^{-1}, x^{0}, x^{\frac{1}{2}}$, and $x^{2}$ when $x=\frac{1}{4}$.
(b) Write $y^{-1}, y^{0}, y^{2}$ and $y^{3}$ in increasing order of size when $y<-1$.

Answer (b) $\qquad$ <. $\qquad$ . $\qquad$ <

18 Write as a single fraction, in its simplest form.

$$
\frac{1-x}{x}-\frac{2+x}{1-2 x}
$$

19

The diagram shows a sector $A O B$ of a circle, centre $O$, radius 9 cm with angle $A O B=50^{\circ}$.
Calculate the area of the segment shaded in the diagram.

2 (a) Find the integer values for $x$ which satisfy the inequality $-3<2 x-1 \leqslant 6$.
(b) Simplify $\frac{x^{2}+3 x-10}{x^{2}-25}$.
(c) (i) Show that $\frac{5}{x-3}+\frac{2}{x+1}=3$ can be simplified to $3 x^{2}-13 x-8=0$.

Answer(c)(i)

(ii) Solve the equation $3 x^{2}-13 x-8=0$.

Show all your working and give your answers correct to two decimal places.

1 Children go to camp on holiday.
(a) Fatima buys bananas and apples for the camp.
(i) Bananas cost $\$ 0.85$ per kilogram.

Fatima buys 20 kg of bananas and receives a discount of $14 \%$.
How much does she spend on bananas?
(ii) Fatima spends $\$ 16.40$ on apples after a discount of $18 \%$.

Calculate the original price of the apples.
Answer(a)(ii) \$
(iii) The ratio number of bananas: number of apples $=4: 5$.

There are 108 bananas.

Calculate the number of apples.
(b) The cost to hire a tent consists of two parts.


The total cost for 4 days is $\$ 27.10$ and for 7 days is $\$ 34.30$.
Write down two equations in $c$ and $d$ and solve them.
(c) The children travel 270 km to the camp, leaving at 0743 and arriving at 1513. Calculate their average speed in $\mathrm{km} / \mathrm{h}$.

Answer(c) ..................................... km/h
[3]
(d) Two years ago $\$ 540$ was put in a savings account to pay for the holiday.

The account paid compound interest at a rate of $6 \%$ per year.
How much is in the account now?


An equilateral 16 -sided figure $A P A^{\prime} Q B \ldots .$. is formed when the square $A B C D$ is rotated $45^{\circ}$ clockwise about its centre to position $A^{\prime} B^{\prime} C^{\prime} D^{\prime}$.
$A B=12 \mathrm{~cm}$ and $A P=x \mathrm{~cm}$.
(a) (i) Use triangle $P A^{\prime} Q$ to explain why $2 x^{2}=(12-2 x)^{2}$.
(ii) Show that this simplifies to $x^{2}-24 x+72=0$.
(iii) Solve $x^{2}-24 x+72=0$. Give your answers correct to 2 decimal places.
(b) (i) Calculate the perimeter of the 16 -sided figure.
(ii) Calculate the area of the 16 -sided figure.


A rectangular-based open box has external dimensions of $2 x \mathrm{~cm},(x+4) \mathrm{cm}$ and $(x+1) \mathrm{cm}$.
(a) (i) Write down the volume of a cuboid with these dimensions.
(ii) Expand and simplify your answer.
(b) The box is made from wood 1 cm thick.
(i) Write down the internal dimensions of the box in terms of $x$.
(ii) Find the volume of the inside of the box and show that the volume of the wood is $8 x^{2}+12 x$ cubic centimetres.
(c) The volume of the wood is $1980 \mathrm{~cm}^{3}$.
(i) Show that $2 x^{2}+3 x-495=0$ and solve this equation.
(ii) Write down the external dimensions of the box.

5 Maria walks 10 kilometres to a waterfall at an average speed of $x$ kilometres per hour.
(a) Write down, in terms of $x$, the time taken in hours.
(b) Maria returns from the waterfall but this time she walks the 10 kilometres at an average speed of $(x+1)$ kilometres per hour. The time of the return journey is 30 minutes less than the time of the first journey.
Write down an equation in $x$ and show that it simplifies to $x^{2}+x-20=0$.
(c) Solve the equation $x^{2}+x-20=0$.
(d) Find the time Maria takes to walk to the waterfall.

7 To raise money for charity, Jalaj walks 22 km , correct to the nearest kilometre, every day for 5 days.
(a) Complete the statement in the answer space for the distance, $d \mathrm{~km}$, he walks in one day.
Answer (a)

$$
\leqslant d<
$$

(b) He raises $\$ 1.60$ for every kilometre that he walks.

Calculate the least amount of money that he raises at the end of the 5 days.

8 Solve the simultaneous equations

$$
\begin{aligned}
& \frac{1}{2} x+2 y=16 \\
& 2 x+\frac{1}{2} y=19
\end{aligned}
$$

Answer $x=$

$$
y=.
$$

9 The wavelength, $w$, of a radio signal is inversely proportional to its frequency, $f$.
When $f=200, w=1500$.
(a) Find an equation connecting $f$ and $w$.

> Answer (a)
(b) Find the value of $f$ when $w=600$.

$$
\text { Answer (b) } f=
$$

13 Solve the equation

$$
\frac{x-2}{4}=\frac{2 x+5}{3}
$$

$$
\text { Answer } x=
$$

14 A company makes two models of television.
Model $A$ has a rectangular screen that measures 44 cm by 32 cm .
Model $B$ has a larger screen with these measurements increased in the ratio 5:4.
(a) Work out the measurements of the larger screen.
(b) Find the fraction $\frac{\text { model } A \text { screen area }}{\text { model } B \text { screen area }}$ in its simplest form.


15 Angharad had an operation costing $\$ 500$.
She was in hospital for $x$ days.
The cost of nursing care was $\$ 170$ for each day she was in hospital.
(a) Write down, in terms of $x$, an expression for the total cost of her operation and nursing care.

$$
\text { Answer }(a) \$
$$

(b) The total cost of her operation and nursing care was $\$ 2370$.

Work out how many days Angharad was in hospital.

5 The length, $y$, of a solid is inversely proportional to the square of its height, $x$.
(a) Write down a general equation for $x$ and $y$.

Show that when $x=5$ and $y=4.8$ the equation becomes $x^{2} y=120$.
(b) Find $y$ when $x=2$.
(c) Find $x$ when $y=10$.
(d) Find $x$ when $y=x$.
(e) Describe exactly what happens to $y$ when $x$ is doubled.
(f) Describe exactly what happens to $x$ when $y$ is decreased by $36 \%$.
(g) Make $x$ the subject of the formula $x^{2} y=120$.
(b)


The diagram shows a right-angled triangle.
The lengths of the sides are given in terms of $y$.
(i) Show that $2 y^{2}-8 y-3=0$.
(ii) Solve the equation $2 y^{2}-8 y-3=0$, giving your answers to 2 decimal places.
(iii) Calculate the area of the triangle.
(b)


NOT TO
SCALE

In the diagram $P Q$ is parallel to $R S$.
$P S$ and $Q R$ intersect at $X$.
$P X=y \mathrm{~cm}, Q X=(y+2) \mathrm{cm}, R X=(2 y-1) \mathrm{cm}$ and $S X=(y+1) \mathrm{cm} \cdot \chi$
(i) Show that $y^{2}-4 y-2=0$.
(ii) Solve the equation $y^{2}-4 y-2=0$.

Show all your working and give your answers correct to two decimal places.
(iii) Write down the length of $R X$.

8 A packet of sweets contains chocolates and toffees.
(a) There are $x$ chocolates which have a total mass of 105 grams.

Write down, in terms of $x$, the mean mass of a chocolate.
(b) There are $x+4$ toffees which have a total mass of 105 grams.

Write down, in terms of $x$, the mean mass of a toffee.
(c) The difference between the two mean masses in parts (a) and (b) is 0.8 grams.

Write down an equation in $x$ and show that it simplifies to $x^{2}+4 x-525=0$.
(d) (i) Factorise $x^{2}+4 x-525$.
(ii) Write down the solutions of $x^{2}+4 x-525=0$.
(e) Write down the total number of sweets in the packet.
$m^{4}-16 n^{4}$ can be written as $\left(m^{2}-k n^{2}\right)\left(m^{2}+k n^{2}\right)$. $k$.

Factorise completely $m^{4} n-16 n^{5}$.

6 (a)


In triangle $A B C$, the line $B D$ is perpendicular to $A C$.
$A D=(x+6) \mathrm{cm}, D C=(x+2) \mathrm{cm}$ and the height $B D=(x+1) \mathrm{cm}$.
The area of triangle $A B C$ is $40 \mathrm{~cm}^{2}$.
(i) Show that $x^{2}+5 x-36=0$.

Answer (a)(i)

(ii) Solve the equation $x^{2}+5 x-36=0$.

$$
\begin{equation*}
\text { Answer(a)(ii) } x= \tag{2}
\end{equation*}
$$

or $x=$
(iii) Calculate the length of $B C$.
(b) Amira takes 9 hours 25 minutes to complete a long walk.
(i) Show that the time of 9 hours 25 minutes can be written as $\frac{113}{12}$ hours.

Answer (b)(i)
(ii) She walks $(3 y+2)$ kilometres at $3 \mathrm{~km} / \mathrm{h}$ and then a further $(y+4)$ kilometres at $2 \mathrm{~km} / \mathrm{h}$.

Show that the total time taken is $\frac{9 y+16}{6}$ hours.
Answer(b)(ii)
(iii) Solve the equation $\frac{9 y+16}{6}=\frac{113}{12}$.

$$
\operatorname{Answer(b)(iii)~} y=
$$

(iv) Calculate Amira's average speed, in kilometres per hour, for the whole walk.


NOT TO
SCALE

A solid metal bar is in the shape of a cuboid of length of 250 cm .
The cross-section is a square of side $x \mathrm{~cm}$.
The volume of the cuboid is $4840 \mathrm{~cm}^{3}$.
(a) Show that $x=4.4$.

Answer (a)
(b) The mass of $1 \mathrm{~cm}^{3}$ of the metal is 8.8 grams. Calculate the mass of the whole metal bar in kilograms.


Answer(b) ............................ k
kg [2]
(c) A box, in the shape of a cuboid measures 250 cm by 88 cm by $h \mathrm{~cm}$.

120 of the metal bars fit exactly in the box.
Calculate the value of $h$.

$$
\begin{equation*}
\text { Answer(c) } h= \tag{2}
\end{equation*}
$$

11 Make $d$ the subject of the formula $c=\frac{5 d+4 w}{2 w}$.

$$
\text { Answer } d=
$$

$12 Q=\{2,4,6,8,10\}$ and $R=\{5,10,15,20\}$.
$15 \in P, \mathrm{n}(P)=1$ and $P \cap Q=\emptyset$.
Label each set and complete the Venn diagram to show this information.


13 Solve the simultaneous equations.

$$
\begin{aligned}
& \frac{2 x+y}{2}=7 \\
& \frac{2 x-y}{2}=17
\end{aligned}
$$

Answer $x=$

$$
y=
$$

9 (a) Solve the following equations.
(i) $\frac{5}{w}=\frac{3}{w+1}$

$$
\operatorname{Answer(a)(i)~w=~}
$$

(ii) $(y+1)^{2}=4$

$$
\operatorname{Answer}(a)(\mathrm{ii}) y=
$$

$\qquad$ or $y=$
(iii) $\frac{x+1}{3}-\frac{x-2}{5}=2$

(b) (i) Factorise $u^{2}-9 u-10$.

## Answer(b)(i)

(ii) Solve the equation $u^{2}-9 u-10=0$.

Answer(b)(ii) $u=$ $\qquad$ or $u=$
(c)


The area of the triangle is equal to the area of the square.
All lengths are in centimetres.
(i) Show that $x^{2}-3 x-2=0$.

## Answer(c)(i)

(ii) Solve the equation $x^{2}-3 x-2=0$, giving your answers correct to 2 decimal places. Show all your working.

$$
\operatorname{Answer}(c)(\mathrm{ii}) x=
$$

$\qquad$ or $x=$
(iii) Calculate the area of one of the shapes.
$\qquad$ $\mathrm{cm}^{2}$

8 (a) $y$ is 5 less than the square of the sum of $p$ and $q$.
Write down a formula for $y$ in terms of $p$ and $q$.

$$
\begin{equation*}
\text { Answer(a) } y= \tag{2}
\end{equation*}
$$

(b) The cost of a magazine is $\$ x$ and the cost of a newspaper is $\$(x-3)$.

The total cost of 6 magazines and 9 newspapers is $\$ 51$.
Write down and solve an equation in $x$ to find the cost of a magazine.

[4]
(c) Bus tickets cost $\$ 3$ for an adult and $\$ 2$ for a child.

There are $a$ adults and $c$ children on a bus.
The total number of people on the bus is 52 .
The total cost of the 52 tickets is $\$ 139$.
Find the number of adults and the number of children on the bus.


9 (a) The cost of a bottle of water is $\$ w$.
The cost of a bottle of juice is $\$ j$.
The total cost of 8 bottles of water and 2 bottles of juice is $\$ 12$.
The total cost of 12 bottles of water and 18 bottles of juice is $\$ 45$.
Find the cost of a bottle of water and the cost of a bottle of juice.

(b) Roshni cycles 2 kilometres at $y \mathrm{~km} / \mathrm{h}$ and then runs 4 kilometres at $(y-4) \mathrm{km} / \mathrm{h}$. The whole journey takes 40 minutes.
(i) Write an equation in $y$ and show that it simplifies to

$$
y^{2}-13 y+12=0 .
$$

Answer(b)(i)
(ii) Factorise $y^{2}-13 y+12$.

Answer(b)(ii)
(iii) Solve the equation $y^{2}-13 y+12=0$.

$$
\operatorname{Answer}(b)(\text { iii }) y=\text {............. } \text { or } y=
$$

(iv) Work out Roshni's running speed.
(c) Solve the equation

$$
u^{2}-u-4=0 .
$$

Show all your working and give your answers correct to 2 decimal placés.


13 (a) Find the value of $x$ when $\frac{18}{24}=\frac{27}{x}$.

$$
\text { Answer(a) } x=
$$

(b) Show that $\frac{2}{3} \div 1 \frac{1}{6}=\frac{4}{7}$.

Write down all the steps in your working.
Answer (b)

14 (a) A drinking glass contains 55 cl of water. Write 55 cl in litres.
(b) The mass of grain in a sack is 35 kg . The grain is divided equally into 140 bags.

Calculate the mass of grain in each bag.
Give your answer in grams.

Answer(b)
g [2]

15 (a) Write 67.499 correct to the nearest integer.

Answer(a)
(b) Write 0.003040506 correct to 3 significant figures.

Answer(b)
(c) $d=56.4$, correct to 1 decimal place.

Write down the lower bound of $d$.

Answer (c)
[1]

10 The cost of a cup of tea is $t$ cents.
The cost of a cup of coffee is $(t+5)$ cents.
The total cost of 7 cups of tea and 11 cups of coffee is 2215 cents.
Find the cost of one cup of tea.

## Answer

cents

11 The volume of a solid varies directly as the cube of its length.
When the length is 3 cm , the volume is $108 \mathrm{~cm}^{3}$.
Find the volume when the length is 5 cm .
$\qquad$ $\mathrm{cm}^{3}$ [3]

16 Write $\frac{2}{x-2}+\frac{3}{x+2}$ as a single fraction.
Give your answer in its simplest form.

17


The diagrams show two mathematically similar containers.
The larger container has a base with diameter 9 cm and a height 20 cm .
The smaller container has a base with diameter $d \mathrm{~cm}$ and a height 10 cm .
(a) Find the value of $d$.

$$
\begin{equation*}
\text { Answer(a) } d= \tag{1}
\end{equation*}
$$

(b) The larger container has a capacity of 1600 ml .

Calculate the capacity of the smaller container.

3 (a)

$$
x=3 m-k
$$

Find the value of
(i) $x$ when $m=2$ and $k=-4$,
Answer(a)(i)
(ii) $m$ when $x=19$ and $k=5$.

## Answer(a)(ii)

(b) Expand the brackets.
(c) Factorise completely.

$$
18 h^{2}-12 h j
$$

Answer(b)
(d) Make $m$ the subject of the formula.

$$
t=8 m+15
$$

Answer(d) $m=$
(e) Solve the equation.

$$
p+3=3(p-5)
$$

7 (a) Solve the equations.
(i) $2 x+3=15-x$

$$
\begin{equation*}
\text { Answer(a)(i) } x= \tag{2}
\end{equation*}
$$

(ii) $\frac{2 y-1}{3}=7$

(iii) $2=\frac{1}{u-1}$

$$
\text { Answer(a)(iii) } u=
$$

(b) Write down equations to show the following.
(i) $p$ is equal to $r$ plus two times $q$.

> Answer(b)(i)
(ii) $k$ is equal to the square of the sum of $l$ and $m$.
Answer(b)(ii)
(c) Pierre walks for 2 hours at $w \mathrm{~km} / \mathrm{h}$ and then for another 3 hours at $(w-1) \mathrm{km} / \mathrm{h}$.

The total distance of Pierre's journey is 11.5 km .
Find the value of $w$.


5 (a) Solve $9<3 n+6 \leqslant 21$ for integer values of $n$.

> Answer(a)
(b) Factorise completely.
(i) $2 x^{2}+10 x y$

Answer(b)(i)
(ii) $3 a^{2}-12 b^{2}$
Answer(b)(ii)
(c)

(ii) Factorise $x^{2}+17 x-168$.

## Answer(c)(ii)

(iii) Solve $x^{2}+17 x-168=0$.
(d) Solve

$$
\frac{15-x}{2}=3-2 x .
$$

(e) Solve $2 x^{2}-5 x-6=0$.

Show all your working and give your answers correct to 2 decimal places.


Answer(e) $x=$
or $x=$

3


The diagram shows a square of side $(x+5) \mathrm{cm}$ and a rectangle which measures $2 x \mathrm{~cm}$ by $x \mathrm{~cm}$.
The area of the square is $1 \mathrm{~cm}^{2}$ more than the area of the rectangle.
(a) Show that $x^{2}-10 x-24=0$.

## Answer (a)

(b) Find the value of $x$.

Answer(b) $x=$
(c) Calculate the acute angle between the diagonals of the rectangle.

(c) Erik runs a race at an average speed of $x \mathrm{~m} / \mathrm{s}$.

His time is $(3 x-9)$ seconds and the race distance is $\left(2 x^{2}-8\right)$ metres.
(i) Write down an equation in $x$ and show that it simplifies to

$$
\begin{equation*}
x^{2}-9 x+8=0 \tag{2}
\end{equation*}
$$

(ii) Solve $x^{2}-9 x+8=0$.
(iii) Write down Erik's time and the race distance.

17 Solve the equation

$$
x^{2}+4 x-22=0 .
$$

Give your answers correct to 2 decimal places.


8 (a) (i) The cost of a book is $\$ x$.
Write down an expression in terms of $x$ for the number of these books which are bought for $\$ 40$.
(ii) The cost of each book is increased by $\$ 2$.

The number of books which are bought for $\$ 40$ is now one less than before.
Write down an equation in $x$ and show that it simplifies to $x^{2}+2 x-80=0$.
(iii) Solve the equation $x^{2}+2 x-80=0$.
(iv) Find the original cost of one book.
(b) Magazines cost $\$ m$ each and newspapers cost $\$ n$ each.

One magazine costs $\$ 2.55$ more than one newspaper.
The cost of two magazines is the same as the cost of five newspapers.
(i) Write down two equations in $m$ and $n$ to show this information.
(ii) Find the values of $m$ and $n$.

1 Two quantities $c$ and $d$ are connected by the formula $c=2 d+30$.
Find $c$ when $d=-100$

2
(a)

$$
\frac{2}{3}+\frac{5}{6}=\frac{x}{2} .
$$

Find the value of $x$.

Answer(a) $x=$
(b)

$$
\frac{5}{3} \div \frac{3}{y}=\frac{40}{9}
$$

Find the value of $y$.

3 Use your calculator to work out
(a) $\sqrt{ }\left(7+6 \times 243^{0.2}\right)$,

> Answer(a)
(b) $2-\tan 30^{\circ} \times \tan 60^{\circ}$.

4 Angharad sleeps for 8 hours each night, correct to the nearest 10 minutes.
The total time she sleeps in the month of November ( 30 nights) is $T$ hours.
Between what limits does $T$ lie?

Answer
$\leqslant T<$

16

$A B C D$ is a trapezium.
(a) Find the area of the trapezium in terms of $x$ and simplify your answer.
Answer(a) .......................................cm²
(b) Angle $B C D=y^{\circ}$. Calculate the value of $y$.

17 Solve the equations
(a) $0.2 x-3=0.5 x$,

$$
\text { Answer(a) } x=
$$

(b) $2 x^{2}-11 x+12=0$.
$\qquad$ or $x=$ $\qquad$

(a) (i) Write down an expression for the area of rectangle $R$.

Answer(a) (i) ................................ $\mathrm{cm}^{2}$
(ii) Show that the total area of rectangles $R$ and $Q$ is $5 x^{2}+30 x+24$ square centimetres.
(b) The total area of rectangles $R$ and $Q$ is $64 \mathrm{~cm}^{2}$.

Calculate the value of $x$ correct to 1 decimal place.

(a) When the area of triangle $A B C$ is $48 \mathrm{~cm}^{2}$,
(i) show that $x^{2}+4 x-96=0$,
(ii) solve the equation $x^{2}+4 x-96=0$,
(iii) write down the length of $A B$.
(b) When $\tan y=\frac{1}{6}$, find the value of $x$.
(c) When the length of $A C$ is 9 cm ,
(i) show that $2 x^{2}+8 x-65=0$,
(ii) solve the equation $2 x^{2}+8 x-65=0$,
(Show your working and give your answers correct to 2 decimal places.)
(iii) calculate the perimeter of triangle $A B C$.


SCALE


The diagram shows two rectangles $A B C D$ and $P Q R S$.
$A B=(2 x+5) \mathrm{cm}, A D=(x+3) \mathrm{cm}, P Q=(x+4) \mathrm{cm}$ and $P S=x \mathrm{~cm}$.
(a) For one value of $x$, the area of rectangle $A B C D$ is $59 \mathrm{~cm}^{2}$ more than the area of rectangle $P Q R S$.
(i) Show that $x^{2}+7 x-44=0$.

Answer(a)(i)
(ii) Factorise $x^{2}+7 x-44$

(iii) Solve the equation $x^{2}+7 x-44=0$.

$$
\text { Answer(a)(iii) } x=
$$

$\qquad$ or $x=$ $\qquad$
(iv) Calculate the size of angle $D B A$.

$$
\begin{equation*}
\text { Answer(a)(iv) Angle } D B A= \tag{2}
\end{equation*}
$$

(b) For a different value of $x$, the rectangles $A B C D$ and $P Q R S$ are similar.
(i) Show that this value of $x$ satisfies the equation $x^{2}-2 x-12=0$. Answer(b)(i)
(ii) Solve the equation $x^{2}-2 x-12=0$, giving your answers correct to 2 decimal places.

$$
\text { Answer(b)(ii) } x=
$$

$$
\text { or } x=
$$

$\qquad$
(iii) Calculate the perimeter of the rectangle $P Q R S$.

9 (a) Solve the equation $\frac{m-3}{4}+\frac{m+4}{3}=-7$.

$$
\text { Answer(a) } m=
$$

(b) (i) $y=\frac{3}{x-1}-\frac{2}{x+3}$

Find the value of $y$ when $x=5$.
(ii) Write $\frac{3}{x-1}-\frac{2}{x+3}$ as a single fraction.
$\square$
(iii) Solve the equation $\frac{3}{x-1}-\frac{2}{x+3}=\frac{1}{x}$.
(c)
$\square$

12 The side of a square is 6.3 cm , correct to the nearest millimetre.
The lower bound of the perimeter of the square is $u \mathrm{~cm}$ and the upper bound of the perimeter is $v \mathrm{~cm}$. Calculate the value of
(a) $u$,

$$
\begin{equation*}
\text { Answer(a) } u= \tag{1}
\end{equation*}
$$

(b) $v-u$.

$$
\operatorname{Answer(b)~} v-u=
$$

$13 a \times 10^{7}+b \times 10^{6}=c \times 10^{6}$
Find $c$ in terms of $a$ and $b$.
Give your answer in its simplest form.

$$
\text { Answer } c=
$$

14 Priyantha completes a 10 km run in 55 minutes 20 seconds.
Calculate Priyantha's average speed in $\mathrm{km} / \mathrm{h}$.

24 (a) Write $\frac{1}{y}-\frac{2}{x}$ as a single fraction in its lowest terms.
(b) Write $\frac{x^{2}+x}{3 x+3}$ in its lowest terms.

25 f: $x \rightarrow 2 x-7$

$$
\mathrm{g}: x \rightarrow \frac{1}{x}
$$

Find
(a) $\mathrm{fg}\left(\frac{1}{2}\right)$,

(b) $\mathrm{gf}(x)$,

$$
\begin{equation*}
\text { Answer(b) } \operatorname{gf}(x)= \tag{1}
\end{equation*}
$$

(c) $\mathrm{f}^{-1}(x)$.

$$
\begin{equation*}
\operatorname{Answer}(c) \mathrm{f}^{-1}(x)= \tag{2}
\end{equation*}
$$



A farmer makes a rectangular enclosure for his animals.
He uses a wall for one side and a total of 72 metres of fencing for the other three sides.
The enclosure has width $x$ metres and area $A$ square metres.
(a) Show that $A=72 x-2 x^{2}$.

Answer (a)

(b) Factorise completely $72 x-2 x^{2}$.
(c) Complete the table for $A=72 x-2 x^{2}$.

| $x$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ | 0 | 310 | 520 |  |  | 550 | 360 |  |

(d) Draw the graph of $A=72 x-2 x^{2}$ for $0 \leqslant x \leqslant 35$ on the grid opposite.

5 (a)


In the right-angled triangle $A B C, A B=x \mathrm{~cm}, B C=(x+7) \mathrm{cm}$ and $A C=17 \mathrm{~cm}$.
(i) Show that $x^{2}+7 x-120=0$.

Answer(a)(i)
(ii) Factorise $x^{2}+7 x-120$.

Answer(a)(ii)
(iii) Write down the solutions of $x^{2}+7 x-120=0$.

$$
\begin{equation*}
\text { Answer(a)(iii) } x=\ldots . . . . . . . . . . . . . . . \quad \text { or } x= \tag{1}
\end{equation*}
$$

(iv) Write down the length of $B C$.
(b)


The rectangle and the square shown in the diagram above have the same area.
(i) Show that $2 x^{2}-15 x-9=0$.

Answer(b)(i)
(ii) Solve the equation $2 x^{2}-15 x-9=0$.

Show all your working and give your answers correct to 2 decimal places.

$$
\text { Answer(b)(ii) } x=\quad . . . . . . . . . . . . . . . . \quad \text { or } x=
$$

$\qquad$
(iii) Calculate the perimeter of the square.
(d) Solve the equation.

$$
2 x^{2}+5 x+1=0
$$

Show all your working and give your answers correct to 2 decimal places.


15 (a) Factorise $t^{2}-4$.

Answer (a)
(b) Factorise completely $a t^{2}-4 a+2 t^{2}-8$.

Answer (b)

16


A set of Russian dolls is made so that the volume, $V$, of each of themaries directly as the cube of its height, $h$.
The doll with a height of 3 cm has a volume of $6.75 \mathrm{~cm}^{3}$.
(a) Find an equation for $V$ in terms of $h$.

$$
\text { Answer (a) } V=
$$

(b) Find the volume of a doll with a height of 2.5 cm .

## EXTENDED MATHEMATICS 2002-2011

CLASSIFIEDS DSTGRAPHS


15 A container ship travelled at $14 \mathrm{~km} / \mathrm{h}$ for 8 hours and then slowed down to $9 \mathrm{~km} / \mathrm{h}$ over a period of 30 minutes.

It travelled at this speed for another 4 hours and then slowed to a stop over 30 minutes.
The speed-time graph shows this voyage.

(a) Calculate the total distance travelled by the ship.

> Answer(a)
(b) Calculate the average speed of the ship for the whole voyage.


A small car accelerates from $0 \mathrm{~m} / \mathrm{s}$ to $40 \mathrm{~m} / \mathrm{s}$ in 6 seconds and then travels at this constant speed. A large car accelerates from $0 \mathrm{~m} / \mathrm{s}$ to $40 \mathrm{~m} / \mathrm{s}$ in 10 seconds.

Calculate how much further the small car travels in the first 10 seconds.


23


The diagram shows the speed-time graph for the first 15 minutes of a train journey.
The train accelerates for 5 minutes and then continues at a constant speed of 40 metres/second.
(a) Calculate the acceleration of the train during the first 5 minutes.

Give your answer in $\mathrm{m} / \mathrm{s}^{2}$.
(b) Calculate the average speed for the first 15 minutes of the train journey.

Give your answer in $\mathrm{m} / \mathrm{s}$.

21 A cyclist is training for a competition and the graph shows one part of the training.

(a) Calculate the acceleration during the first 10 seconds.

Answer(a)
$\mathrm{m} / \mathrm{s}^{2}[2]$
(b) Calculate the distance travelled in the first 30 seconds.

Answer(b)
m [2]
(c) Calculate the average speed for the entire 45 seconds.

1 (a) A train completed a journey of 850 kilometres with an average speed of 80 kilometres per hour. Calculate, giving exact answers, the time taken for this journey in
(i) hours,
(ii) hours, minutes and seconds.
(b) Another train took 10 hours 48 minutes to complete the same 850 km journey.
(i) It departed at 1920 .

At what time, on the next day, did this train complete the journey?
(ii) Calculate the average speed, in kilometres per hour, for the journey.
(c)


The solid line $O A B C D$ on the grid shows the first 10 seconds of a car journey.
(i) Describe briefly what happens to the speed of the car between $B$ and $C$.
(ii) Describe briefly what happens to the acceleration of the car between $B$ and $C$.
(iii) Calculate the acceleration between $A$ and $B$.
(iv) Using the broken straight line $O C$, estimate the total distance travelled by the car in the whole 10 seconds.
(v) Explain briefly why, in this case, using the broken line makes the answer to part (iv) a good estimate of the distance travelled.
(vi) Calculate the average speed of the car during the 10 seconds. Give your answer in kilometres per hour.


The diagram shows part of a journey by a truck.
(a) The truck accelerates from rest to $18 \mathrm{~m} / \mathrm{s}$ in 30 seconds.

Calculate the acceleration of the truck.

Answer(a)
$\mathrm{m} / \mathrm{s}^{2}$
(b) The truck then slows down in 10 seconds for some road works and travels through the road works at $12 \mathrm{~m} / \mathrm{s}$.
At the end of the road works it accelerates back to a speed of $18 \mathrm{~m} / \mathrm{s}$ in 10 seconds.
Find the total distance travelled by the truck in the 100 seconds.

16 The graphs show the speeds of two cyclists, Alonso and Boris.
Alonso accelerated to $10 \mathrm{~m} / \mathrm{s}$, travelled at a steady speed and then slowed to a stop.


Boris accelerated to his maximum speed, $v \mathrm{~m} / \mathrm{s}$, and then slowed to a stop.


Both cyclists travelled the same distance in the 16 seconds.
Calculate the maximum speed for Boris.
Show all your working.

19 The braking distance, $d$ metres, for Alex's car travelling at $v \mathrm{~km} / \mathrm{h}$ is given by the formula

$$
200 d=v(v+40) .
$$

(a) Calculate the missing values in the table.

| $v$ <br> $(\mathrm{~km} / \mathrm{h})$ | 0 | 20 | 40 | 60 | 80 | 100 | 120 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d$ <br> (metres) | 0 |  | 16 |  | 48 |  | 96 |

(b) On the grid below, draw the graph of $200 d=v(v+40)$ for $0 \leqslant v \leqslant 120$.

(c) Find the braking distance when the car is travelling at $110 \mathrm{~km} / \mathrm{h}$.

Answer(c)
m
[1]
(d) Find the speed of the car when the braking distance is 80 m .


The diagram shows the speed-time graph for 15 seconds of the journey of a cyclist.
(a) Calculate the acceleration of the cyclist during the first 4 seconds.
(b) Calculate the average speed for the first 15 seconds.

Answer (a) ........................... m/s ${ }^{2}$



20


The graph shows part of Ali's journey from home to his school.
The school is 900 m from his home.
He walks 200 m to his friend's house and waits there.
He then takes 20 minutes to walk with his friend to their school.
(a) Complete the travel graph showing Ali's journey
(b) How long does he wait at his friend's house?
(c) Calculate the average speed for Ali's complete journey from home to his school.

Give your answer in kilometres per hour.

19


The diagram shows the speed-time graph of a train journey between two stations.
The train accelerates for two minutes, travels at a constant maximum speed, then slows to a stop.
(a) Write down the number of seconds that the train travels at its constant maximum speed.
(b) Calculate the distance between the two stations in metres.

Answer(b)
m [3]
(c) Find the acceleration of the train in the first two minutes.

Give your answer in $\mathbf{m} / \mathbf{s}^{2}$.

Question 20 is printed on the next page.

22


A train journey takes one hour.
The diagram shows the speed-time graph for this journey.
(a) Calculate the total distance of the journey.

Give your answer in kilometres.
(b) (i) Convert 3 kilometres/minute into metres/second.

> Answer(b)(i)
(ii) Calculate the acceleration of the train during the first 4 minutes.

Give your answer in metres/second ${ }^{2}$.

4


Sonia travels from home to the library.
She walks to the bus stop and waits for a bus to take her to the library.
(a) Write down
(i) the distance to the bus stop,

Answer(a)(i) ............................... km [1]
(ii) how many minutes Sonia waits for a bus,
(iii) how many minutes the bus journey takes to the library.
(b) Calculate, in kilometres per hour,
(i) Sonia's walking speed,

Answer(b)(i)
$\mathrm{km} / \mathrm{h}$ [1]
(ii) the speed of the bus,

Answer(b)(ii)
(iii) the average speed for Sonia's journey from home to the library.
(c) Sonia works in the library for one hour.

Then she travels home by car.
The average speed of the car is $30 \mathrm{~km} / \mathrm{h}$.
Complete the travel graph.

13


Ameni is cycling at 4 metres per second.
After 3.5 seconds she starts to decelerate and after a further 2.5 seconds she stops. The diagram shows the speed-time graph for Ameni.
Calculate
(a) the constant deceleration,

> Answer (a) ...@......................................m/s²
(b) the total distance travelled during the 6 seconds.

Distance from
Hilltown (km)


The graph shows the distance, in kilometres, of a train from Hilltown.
Find the speed of the train in kilometres per hour at
(a) 0830 ,

> Answer(a)
$\qquad$ km/h [2]
(b) 0900 .

> Answer(b) km/h [1]

20


An athlete, in a race, accelerates to a speed of 12.4 metres per second in 3 seconds.
He runs at this speed for the next 5 seconds and slows down over the last 2 seconds as shown in the speed-time graph above.
He crosses the finish line after 10 seconds.
The total distance covered is 100 m .
(a) Calculate the distance he runs in the first 8 seconds.

(b) Calculate his speed when he crosses the finish line.


The graph shows the speed of a truck and a car over 60 seconds.
(a) Calculate the acceleration of the car over the first 45 secomds.

$\mathrm{m} / \mathrm{s}^{2}$
(b) Calculate the distance travelled by the car while it was travelling faster than the truck.

21 An animal starts from rest and accelerates to its top speed in 7 seconds. It continues at this speed for 9 seconds and then slows to a stop in a further 4 seconds.

The graph shows this information.

(a) Calculate its acceleration during the first seven seconds.
(b) Write down its speed 18 seconds after the start.

Answer(b) $\qquad$ $\mathrm{m} / \mathrm{s}$
(c) Calculate the total distance that the animal travelled.


The graph shows 40 seconds of a car journey
The car travelled at a constant speed of $20 \mathrm{~m} / \mathrm{s}$, decelerated to $8 \mathrm{~m} / \mathrm{s}$ then accelerated back to $20 \mathrm{~m} / \mathrm{s}$.
Calculate
(a) the deceleration of the car,

Answer(a) $\qquad$ $\mathrm{m} / \mathrm{s}^{2}$
(b) the total distance travelled by the car during the 40 seconds.

12


A car starts from rest. The speed-time graph shows the first 7 seconds of its journey.
Calculate
(a) the acceleration between 2 and 7 seconds,
$\qquad$
Answer (a)
$\mathrm{m} / \mathrm{s}^{2}$
(b) the distance travelled by the car during the first 7 seconds.

# EXTENDED MATHEMATICS 2002-2011 <br> INEQUALITIESnLP 

Compiled \& Edited

Muhammad Maaz Rashid

14


The region $R$ is bounded by three lines.
Write down the three inequalities which define the region $R$.

Answer $\qquad$
$\qquad$

10 Hassan stores books in large boxes and small boxes.
Each large box holds 20 books and each small box holds 10 books.
He has $x$ large boxes and $y$ small boxes.
(a) Hassan must store at least 200 books.

Show that $2 x+y \geqslant 20$.
Answer(a)
(b) Hassan must not use more than 15 boxes.

He must use at least 3 small boxes.
The number of small boxes must be less than or equal to the number of large boxes.
Write down three inequalities to show this information.

(c) On the grid, show the information in part (a) and part (b) by drawing four straight lines and shading the unwanted regions.

(d) A large box costs $\$ 5$ and a small box costs $\$ 2$.
(i) Find the least possible total cost of the boxes.

Answer(d)(i) \$
(ii) Find the number of large boxes and the number of small boxes which give this least possible cost.


22

(a) Find the equation of the line $l$ shown in the grid above.

(b) Write down three inequalities which define the region $R$.

Answer (b) $\qquad$
$\qquad$
$\qquad$
$\qquad$

## 9 Answer all of this question on a sheet of graph paper.

A shop buys $x$ pencils and $y$ pens.
Pencils cost 15 cents each and pens cost 25 cents each.
(a) There is a maximum of $\$ 20$ to spend.

Show that $3 x+5 y \leqslant 400$.
(b) The number of pens must not be greater than the number of pencils.

Write down an inequality, in terms of $x$ and $y$, to show this information.
(c) There must be at least 35 pens.

Write down an inequality to show this information.
(d) (i) Using a scale of 1 cm to represent 10 units on each axis, draw an $x$-axis for $0 \leqslant x \leqslant 150$ and a $y$-axis for $0 \leqslant y \leqslant 100$.
(ii) Draw three lines on your graph to show the inequalities in parts (a), (b) and (c). Shade the unwanted regions.
(e) When 70 pencils are bought, what is the largest possible number of pens?
(f) The profit on each pencil is 5 cents and the profit on each pen is 7 cents. Find the largest possible profit.

## 9 Answer the whole of this question on a sheet of graph paper.

A taxi company has "SUPER" taxis and "MINI" taxis.
One morning a group of 45 people needs taxis.
For this group the taxi company uses $x$ "SUPER" taxis and $y$ "MINI" taxis.
A "SUPER" taxi can carry 5 passengers and a "MINI" taxi can carry 3 passengers.
So $5 x+3 y \geqslant 45$.
(a) The taxi company has 12 taxis.

Write down another inequality in $x$ and $y$ to show this information.
(b) The taxi company always uses at least 4 "MINI" taxis.

Write down an inequality in $y$ to show this information.
(c) Draw $x$ and $y$ axes from 0 to 15 using 1 cm to represent 1 unit on each axis.
(d) Draw three lines on your graph to show the inequality $5 x+3 y \geqslant 45$ and the inequalities from parts
(a) and (b).

Shade the unwanted regions.
(e) The cost to the taxi company of using a "SUPER" taxi is $\$ 20$ and the cost of using a "MINI" taxi is $\$ 10$.
The taxi company wants to find the cheapest way of providing "SUPER" and "MINI" taxis for this group of people.
Find the two ways in which this can be done.
(f) The taxi company decides to use 11 taxis for this group.
(i) The taxi company charges $\$ 30$ for the use of each "SUPER" taxi and $\$ 16$ for the use of each "MINI" taxi.
Find the two possible total charges.
(ii) Find the largest possible profit the company can make, using 11 taxis.

(a) One of the lines in the diagram is labelled $y=m x+c$.

Find the values of $m$ and $c$.
$c=$
(b) Show, by shading all the unwanted regions on the diagram, the region defined by the inequalities

$$
x \geqslant 1, \quad y \leqslant m x+c, \quad y \geqslant x+2 \quad \text { and } \quad y \geqslant 4
$$

Write the letter $\mathbf{R}$ in the region required.


Answer $\qquad$
$\qquad$

## 9 Answer the whole of this question on a sheet of graph paper.

Tiago does some work during the school holidays.
In one week he spends $x$ hours cleaning cars and $y$ hours repairing cycles
The time he spends repairing cycles is at least equal to the time he spends cleaning cars.
This can be written as $y \geqslant x$.
He spends no more than 12 hours working.
He spends at least 4 hours cleaning cars.
(a) Write down two more inequalities in $x$ and/or $y$ to show this information.
(b) Draw $x$ and $y$ axes from 0 to 12 , using a scale of 1 cm to represent 1 unit on each axis.
(c) Draw three lines to show the three inequalities. Shade the unwanted regions.
(d) Tiago receives $\$ 3$ each hour for cleaning cars and $\$ 1.50$ each hour for repairing cycles.
(i) What is the least amount he could receive?
(ii) What is the largest amount he could receive?

(a) Draw the lines $y=2, x+y=6$ and $y=2 x$ on the grid above.
(b) Label the region $R$ which satisfies the three inequalities
$x+y \geqslant 6, \quad y \geqslant 2 \quad$ and $\quad y \leqslant 2 x$.


Find the three inequalities which define the shaded triangle in the diagram.


Answer
...........................................................


23

(a) Find the equations of the lines $l_{1}, l_{2}$ and $l_{3}$.

(b) The unshaded region, labelled $R$, is defined by three inequalities. Write down these three inequalities.

Answer (b) $\qquad$
$\qquad$
$\qquad$

# EXTENDED MATHEMATICS 2002-2011 CLASSIFIEDS NUMBERS 

1 Use your calculator to find $\sqrt{\frac{45 \times 5.75}{3.1+1.5}}$.

> Answer

2 Work out $2\left(3 \times 10^{8}-4 \times 10^{6}\right)$, giving your answer in standard form.

3 Write the following in order of size, largest first.


4 Write down all the working to show that $\frac{\frac{3}{5}+\frac{2}{3}}{\frac{3}{5} \times \frac{2}{3}}=3 \frac{1}{6}$.
Answer

5 A circle has a radius of 50 cm .
(a) Calculate the area of the circle in $\mathrm{cm}^{2}$.

```
Answer(a)
cm [2]
```

(b) Write your answer to part (a) in $\mathrm{m}^{2}$.

1 A bus leaves a port every 15 minutes, starting at 0900. The last bus leaves at 1730 .

How many times does a bus leave the port during one day?

3 Use your calculator to find the value of
(a) $3^{0} \times 2.5^{2}$,
(b) $2.5^{-2}$.


4 The cost of making a chair is $\$ 28$ correct to the nearest dollar.

Calculate the lower and upper bounds for the cost of making 450 chairs.

Answer lower bound \$
upper bound \$

5 Jiwan incorrectly wrote $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=1 \frac{3}{9}$.
Show the correct working and write down the answer as a mixed number.

6 The force, $F$, between two magnets varies inversely as the square of the distance, $d$, between them. $F=150$ when $d=2$.

Calculate $F$ when $d=4$.


$$
\begin{equation*}
\text { Answer } F= \tag{3}
\end{equation*}
$$

11 Find the values of $m$ and $n$.
(a) $2^{m}=0.125$

$$
\text { Answer(a) } m=
$$

(b) $2^{4 n} \times 2^{2 n}=512$

$$
\text { Answer(b) } n=
$$

1 Martha divides $\$ 240$ between spending and saving in the ratio

$$
\text { spending }: \text { saving }=7: 8
$$

Calculate the amount Martha has for spending.

## Answer \$

2
$210 \quad 211$
212
$213 \quad 214$
215
216

From the list of numbers, find
(a) a prime number,
(b) a cube number.

$$
\begin{aligned}
& x+5 y=22 \\
& x+3 y=12
\end{aligned}
$$

$$
\begin{array}{r}
\text { Answer } x= \\
y=
\end{array}
$$

4 Find the value of $\quad\left(\frac{27}{8}\right)^{-\frac{4}{3}}$.
Give your answer as an exact fraction.

5 The population of a city is 128000 , correct to the nearest thousand.
(a) Write 128000 in standard form.

> Answer(a)
(b) Write down the upper bound of the population.

6 Pedro invested $\$ 800$ at a rate of $5 \%$ per year compound interest.
Calculate the total amount he has after 2 years.

> Answer \$
$7 \quad$ Show that $\quad 3^{-2}+2^{-2}=\frac{13}{36}$.
Write down all the steps of your working.
Answer

8 Find the value of $\frac{\sqrt[3]{17.1-1.89}}{10.4+\sqrt{8.36}}$.
Answer

9 In Vienna, the mid-day temperatures, in ${ }^{\circ} \mathrm{C}$, are recorded during a week in December.
This information is shown below.

$$
\begin{array}{lllllll}
-2 & 2 & 1 & -3 & -1 & -2 & 0
\end{array}
$$

## Calculate

(a) the difference between the highest temperature and the lowest temperature,
(b) the mean temperature.

Answer(b)
${ }^{\circ} \mathrm{C}$ [2]

10 Maria decides to increase her homework time of 8 hours per week by $15 \%$.
Calculate her new homework time.
Give your answer in hours and minutes.

12 Alberto changes 800 Argentine pesos (ARS) into dollars (\$) when the rate is $\$ 1=3.8235$ ARS. He spends $\$ 150$ and changes the remaining dollars back into pesos when the rate is $\$ 1=3.8025$ ARS .

Calculate the amount Alberto now has in pesos.


Answer
kg [3]

1 (a) Abdullah and Jasmine bought a car for $\$ 9000$.
Abdullah paid $45 \%$ of the $\$ 9000$ and Jasmine paid the rest.
(i) How much did Jasmine pay towards the cost of the car?
Answer(a)(i) \$
(ii) Write down the ratio of the payments Abdullah: Jasmine in its simplest form.

> Answer(a)(ii) :
(b) Last year it cost $\$ 2256$ to run the car.

Abdullah, Jasmine and their son Henri share this cost in the ratio $8: 3: 1$.
Calculate the amount each paid to run the car.

## Answer(b) Abdullah \$

 Jasmine \$ $\qquad$Henris
(c) (i) A new truck costs $\$ 15000$ and loses $23 \%$ of its yalue each year.

Calculate the value of the truck after three years.

## Answer(c)(i) \$

(ii) Calculate the overall percentage loss of the truck's value after three years.


A rectangular tank measures 1.2 m by 0.8 m by 0.5 m .
(a) Water flows from the full tank into a cylinder at a rate of $0.3 \mathrm{~m}^{3} / \mathrm{min}$

Calculate the time it takes for the full tank to empty.
Give your answer in minutes and seconds.
(b) The radius of the cylinder is 0.4 m .

Calculate the depth of water, $d$, when all the water from the rectangular tank is in the cylinder.

Answer(b)
(c) The cylinder has a height of 1.2 m and is open at the top. The inside surface is painted at a cost of $\$ 2.30$ per $\mathrm{m}^{2}$. Calculate the cost of painting the inside surface.

1 Javed says that his eyes will blink 415000000 times in 79 years.
(a) Write 415000000 in standard form.

> Answer (a)
(b) One year is approximately 526000 minutes.

Calculate, correct to the nearest whole number, the average number of times his eyes will blink per minute.

> Answer (b)

2 Luis and Hans both have their birthdays on January 1st. In 2002 Luis is 13 and Hans is 17 years old.
(a) Which is the next year after 2002 when both their ages will be prime numbers?

## Answer (a)

(b) In which year was Hans twice as old as Luis?

Answer (b)

7 The temperature decreases from $25^{\circ} \mathrm{C}$ to $22^{\circ} \mathrm{C}$.
Calculate the percentage decrease.

9 Elena has eight rods each of length 10 cm , correct to the nearest centimetre.
She places them in the shape of a rectangle, three rods long and one rod wide.


NOT TO
SCALE
(a) Write down the minimum length of her rectangle.

Answer (a) cm
(b) Calculate the minimum area of her rectangle.

> Answer (b)
$\mathrm{cm}^{2}$

10 Mona made a model of a building using a scale of 1:20. The roof of the building had an area of $300 \mathrm{~m}^{2}$.
(a) Calculate the area of the roof of the model in square metres.
$\qquad$Answer (a)
$\mathrm{m}^{2}$
(b) Write your answer in square centimetres.



Two circles have radii $r \mathrm{~cm}$ and $4 r \mathrm{~cm}$.
Find, in terms of $\pi$ and $r$.
(a) the area of the circle with radius $4 r \mathrm{~cm}$,
(b) the area of the shaded ring,
Answer (a) $\qquad$ $\mathrm{cm}^{2}$

## Answer (b)

).. $\mathrm{cm}^{2}$
(c) the total length of the inner and outer edges of the shaded ring.

18 (a) Omar changed 800 rands into dollars when the rate was $\$ 1=6.25$ rands.
(i) How many dollars did Omar receive?

> Answer (a)(i) \$
$\qquad$
(ii) Three months later he changed his dollars back into rands when the rate was $\$ 1=6.45$ rands. How many extra rands did he receive?

> Answer (a)(ii)
rands
(b) Omar's brother invested 800 rands for three months at a simple interest rate of $12 \%$ per year. How much interest did he receive?

1 (a) One day Amit works from 0800 until 1700.
The time he spends on filing, computing, writing and having lunch is in the ratio
Filing: Computing: Writing: Lunch $=2: 5: 4: 1$.
Calculate the time he spends
(i) writing,
(ii) having lunch, giving this answer in minutes.
(b) The amount earned by Amit, Bernard and Chris is in the ratio $2: 5: 3$.

Bernard earns $\$ 855$ per week.
Calculate how much
(i) Amit earns each week,
(ii) Chris earns each week.
(c) After 52 weeks Bernard has saved \$2964.

What fraction of his earnings has he saved?
Give your answer in its lowest terms.
(d) Chris saves $\$ 3500$ this year. This is $40 \%$ more than he saved last year. Calculate how much he saved last year.

1 Write in order of size, smallest first,

$$
\frac{5}{98}, \quad 0.049, \quad 5 \% .
$$

Answer $\qquad$ .<. ...<.

2 The graph below can be used to convert between euros ( $€$ ) and pounds ( $£$ ).

Pounds (£)


Answer (a) $€$
(b) Change $€ 90$ into pounds.

Answer (b) £ [1]

3 The top speed of a car is 54 metres per second. Change this speed into kilometres per hour.

Answer $\qquad$ .km/h

5 The ratios of teachers : male students : female students in a school are 2:17:18.
The total number of students is 665 .
Find the number of teachers.

Answer

6 A rectangular field is 18 metres long and 12 metres wide.
Both measurements are correct to the nearest metre.
Work out exactly the smallest possible area of the field.
$\qquad$ .$m^{2}$

8 Complete this table of squares and cubes.
The numbers are not in sequence.

| Number | Square | Cube |
| :---: | :---: | :---: |
| 3 | 9 | 27 |
| $\ldots \ldots .$. | 121 | $\ldots \ldots .$. |
| $\ldots \ldots .$. | $\ldots \ldots$. | 2744 |
| $\ldots \ldots .$. | $\ldots \ldots .$. | -343 |

15 In 1950, the population of Switzerland was 4714900. In 2000, the population was 7087000 .
(a) Work out the percentage increase in the population from 1950 to 2000.
Answer (a)......................................... \% [2]
(b) (i) Write the 1950 population correct to 3 significant figures.
Answer (b)(i)
(ii) Write the 2000 population in standard form.
Answer (b)(ii).


1 Tickets for the theatre cost either $\$ 10$ or $\$ 16$.
(a) Calculate the total cost of 197 tickets at $\$ 10$ each and 95 tickets at $\$ 16$ each.
(b) On Monday, 157 tickets at $\$ 10$ and $n$ tickets at $\$ 16$ were sold. The total cost was $\$ 4018$. Calculate the value of $n$.
(c) On Tuesday, 319 tickets were sold altogether. The total cost was $\$ 3784$. Using $x$ for the number of $\$ 10$ tickets sold and $y$ for the number of $\$ 16$ tickets sold, write down two equations in $x$ and $y$.

Solve your equations to find the number of $\$ 10$ tickets and the number of $\$ 16$ tickets sold.
(d) On Wednesday, the cost of a $\$ 16$ ticket was reduced by $15 \%$. Calculate this new reduced cost.
(e) The $\$ 10$ ticket costs $25 \%$ more than it did last year. Calculate the cost last year.


1 A train left Sydney at 2320 on December $18^{\text {th }}$ and arrived in Brisbane at 0240 on December $19^{\text {th }}$. How long, in hours and minutes, was the journey?

Answer $\qquad$ h $\min [1]$

2 Use your calculator to find the value of

$$
\begin{aligned}
& \frac{6 \sin 50^{\circ}}{\sin 25^{\circ}} . \\
& \text { Answer }
\end{aligned}
$$

3 Write the numbers $0.5^{2}, \sqrt{0.5}, 0.5^{3}$ in order with the smallest first.

Answer


4 Simplify


6 The population, $P$, of a small island was 6380 , correct to the nearest 10 . Complete the statement about the limits of $P$.

7 Work out the value of

$$
\frac{-\frac{1}{2}-\frac{3}{8}}{-\frac{1}{2}+\frac{3}{8}} .
$$

9 Sara has $\$ 3000$ to invest for 2 years.
She invests the money in a bank which pays simple interest at the rate of $7.5 \%$ per year.
Calculate how much interest she will have at the end of the 2 years.

10 The area of a small country is 78133 square kilometres.
(a) Write this area correct to 1 significant figure.

$$
\text { Answer(a) ................................................... } \mathrm{km}^{2} \text { [1] }
$$

(b) Write your answer to part (a) in standard form.

1 Fatima and Mohammed each buys a bike.
(a) Fatima buys a city-bike which has a price of $\$ 120$.

She pays $60 \%$ of this price and then pays $\$ 10$ per month for 6 months.
(i) How much does Fatima pay altogether?
(ii) Work out your answer to part (a)(i) as a percentage of the original price of $\$ 120$.
(b) Mohammed pays $\$ 159.10$ for a mountain-bike in a sale.

The original price had been reduced by $14 \%$.
Calculate the original price of the mountain-bike.
(c) Mohammed's height is 169 cm and Fatima's height is 156 cm .

The frame sizes of their bikes are in the same ratio as their heights.
The frame size of Mohammed's bike is 52 cm .
Calculate the frame size of Fatima's bike.
(d) Fatima and Mohammed are members of a school team which takes part in a bike ride for charity.
(i) Fatima and Mohammed ride a total distance of 36 km .

The ratio distance Fatima rides : distance Mohammed rides is $11: 9$.
Work out the distance Fatima rides.
(ii) The distance of 36 km is only $\frac{2}{23}$ of the total distance the team ridés.

Calculate this total distance.

## 1 Calculate

$$
\frac{5^{2}}{2^{5}}
$$

(a) giving your answer as a fraction,

> Answer (a)
(b) giving your answer as a decimal.

Answer (b)


A shop has a wheelchair ramp to its entrance from the pavement. The ramp is 3.17 metres long and is inclined at $5^{\circ}$ to the horizontal. Calculate the height, $h$ metres, of the entrance above the pavement. Show all your working.

3 A block of cheese, of mass 8 kilograms, is cut by a machine into 500 equal slices.
(a) Calculate the mass of one slice of cheese in kilograms.
Answer (a)
(b) Write your answer to part (a) in standard form.

7 To raise money for charity, Jalaj walks 22 km , correct to the nearest kilometre, every day for 5 days.
(a) Complete the statement in the answer space for the distance, $d \mathrm{~km}$, he walks in one day.
Answer (a)

$$
\leqslant d<
$$

(b) He raises $\$ 1.60$ for every kilometre that he walks.

Calculate the least amount of money that he raises at the end of the 5 days.

Answer (b) \$

8 Solve the simultaneous equations

$$
\begin{aligned}
& \frac{1}{2} x+2 y=16 \\
& 2 x+\frac{1}{2} y=19
\end{aligned}
$$

Answer $x=$

$$
y=.
$$

9 The wavelength, $w$, of a radio signal is inversely proportional to its frequency, $f$.
When $f=200, w=1500$.
(a) Find an equation connecting $f$ and $w$.

> Answer (a)
(b) Find the value of $f$ when $w=600$.

$$
\text { Answer (b) } f=
$$

1 Hassan sells fruit and vegetables at the market.
(a) The mass of fruit and vegetables he sells is in the ratio

$$
\text { fruit : vegetables }=5: 7
$$

Hassan sells 1.33 tonnes of vegetables.
How many kilograms of fruit does he sell?
(b) The amount of money Hassan receives from selling fruit and vegetables is in the ratio
fruit : vegetables $=9: 8$.
Hassan receives a total of $\$ 765$ from selling fruit and vegetables.
Calculate how much Hassan receives from selling fruit.
(c) Calculate the average price of Hassan's fruit, in dollars per kilogram.
(d) (i) Hassan sells oranges for $\$ 0.35$ per kilogram.

He reduces this price by $40 \%$.
Calculate the new price per kilogram.
(ii) The price of $\$ 0.35$ per kilogram of oranges is an increase of $25 \%$ on the previous day's price. Calculate the previous day's price.

1 The planet Neptune is 4496000000 kilometres from the Sun.
Write this distance in standard form.

> Answer

2 Write down the next prime number after 89.

> Answer

3 The table gives the average surface temperature $\left({ }^{\circ} \mathrm{C}\right)$ on the following planets.

| Planet | Earth | Mercury | Neptune | Pluto | Saturn | Uranus |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average temperature | 15 | 350 | -220 | -240 | -180 | -200 |

(a) Calculate the range of these temperatures.

Answer (a)

(b) Which planet has a temperature $20^{\circ} \mathrm{C}$ lower than that of Uranus?

4 Work out

5 In triangle $A B C, A B=6 \mathrm{~cm}, A C=8 \mathrm{~cm}$ and $B C=12 \mathrm{~cm}$. Angle $A C B=26.4^{\circ}$.
Calculate the area of the triangle $A B C$.

$\qquad$

10 For the sequence $\quad 5 \frac{1}{2}, \quad 7, \quad 8 \frac{1}{2}, \quad 10, \quad 11 \frac{1}{2}$,
(a) find an expression for the $n$th term,

Answer(a)
[2]
(b) work out the 100th term.

## Answer(b)

11

$$
\mathrm{f}(x)=\frac{x+3}{x}, \quad x \neq 0
$$

(a) Calculate $\mathrm{f}\left(\frac{1}{4}\right)$.
(b) Solve $\mathrm{f}(x)=\frac{1}{4}$.
( $x$.


Answer(b) $x=$

12 Solve the simultaneous equations

$$
\begin{aligned}
& 0.4 x+2 y=10 \\
& 0.3 x+5 y=18
\end{aligned}
$$

$$
y=
$$

13 Solve the equation

$$
\frac{x-2}{4}=\frac{2 x+5}{3}
$$

$$
\text { Answer } x=
$$

14 A company makes two models of television.
Model $A$ has a rectangular screen that measures 44 cm by 32 cm .
Model $B$ has a larger screen with these measurements increased in the ratio 5:4.
(a) Work out the measurements of the larger screen.
(b) Find the fraction $\frac{\text { model } A \text { screen area }}{\text { model } B \text { screen area }}$ in its simplest form.


15 Angharad had an operation costing $\$ 500$.
She was in hospital for $x$ days.
The cost of nursing care was $\$ 170$ for each day she was in hospital.
(a) Write down, in terms of $x$, an expression for the total cost of her operation and nursing care.

$$
\text { Answer }(a) \$
$$

(b) The total cost of her operation and nursing care was $\$ 2370$.

Work out how many days Angharad was in hospital.

16 In 2004 Colin had a salary of $\$ 7200$.
(a) This was an increase of $20 \%$ on his salary in 2002.

Calculate his salary in 2002.

Answer(a)\$
(b) In 2006 his salary increased to $\$ 8100$.

Calculate the percentage increase from 2004 to 2006.

## Answer(b) <br> \% [2]

$17 \mathrm{n}(A)=18, \mathrm{n}(B)=11$ and $\mathrm{n}(A \cup B)^{\prime}=0$.
(a) Label the Venn diagram to show the sets $A$ and $B$ where $\mathrm{n}(A \cup B)=18$.

Write down the number of elements in each region.

(b) Draw another Venn diagram to show the sets $A$ and $B$ where $\mathrm{n}(A \cup B)=29$.

Write down the number of elements in each region.


22


Kalid and his brother have $\$ 2000$ each to invest for 3 years.
(a) North Eastern Bank advertises savings with simple interest at 5\% per year.

Kalid invests his money in this bank.
How much money will he have at the end of 3 years?

$$
\text { Answer }(a) \$
$$

(b) South Western Bank advertises savings with compound interest at 4.9\% per year.

Kalid's brother invests his money in this bank.
At the end of 3 years, how much more money will he have than Kalid?


The largest possible circle is drawn inside a semicircle, as shown in the diagram.
The distance $A B$ is 12 centimetres.
(a) Find the shaded area.
(b) Find the perimeter of the shaded area.
cm [2]

1 (a) The scale of a map is 1:20000000.
On the map, the distance between Cairo and Addis Ababa is 12 cm .
(i) Calculate the distance, in kilometres, between Cairo and Addis Ababa.
(ii) On the map the area of a desert region is 13 square centimetres.

Calculate the actual area of this desert region, in square kilometres.
(b) (i) The actual distance between Cairo and Khartoum is 1580 km .

On a different map this distance is represented by 31.6 cm .
Calculate, in the form $1: n$, the scale of this map.
(ii) A plane flies the 1580 km from Cairo to Khartoum.

It departs from Cairo at 1155 and arrives in Khartoum at 1403.
Calculate the average speed of the plane, in kilometres per hour.

1 Vreni took part in a charity walk.
She walked a distance of 20 kilometres.
(a) She raised money at a rate of $\$ 12.50$ for each kilometre.
(i) How much money did she raise by walking the 20 kilometres?
(ii) The money she raised in part (a)(i) was $\frac{5}{52}$ of the total money raised.

Work out the total money raised.
(iii) In the previous year the total money raised was $\$ 2450$.

Calculate the percentage increase on the previous year's total.
(b) Part of the 20 kilometres was on a road and the rest was on a footpath.

The ratio road distance : footpath distance was 3:2.
(i) Work out the road distance.
(ii) Vreni walked along the road at $3 \mathrm{~km} / \mathrm{h}$ and along the footpath at $2.5 \mathrm{~km} / \mathrm{h}$.

How long, in hours and minutes, did Vreni take to walk the 20 kilometres?
(iii) Work out Vreni's average speed.
(iv) Vreni started at 0855 . At what time did she finish?
(c) On a map, the distance of 20 kilometres was represented by a length of 80 centimetres.

The scale of the map was $1: n$. Calculate the value of $n$.

8 Answer the whole of this question on a sheet of graph paper.
Use one side for your working and one side for your graphs.

Alaric invests \$100 at 4\% per year compound interest.
(a) How many dollars will Alaric have after 2 years?
(b) After $x$ years, Alaric will have $y$ dollars.

He knows a formula to calculate $y$.
The formula is $y=100 \times 1.04^{x}$

| $x$ (Years) | 0 | 10 | 20 | 30 | 40 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ (Dollars) | 100 | $p$ | 219 | $q$ | 480 |

Use this formula to calculate the values of $p$ and $q$ in the table.
(c) Using a scale of 2 cm to represent 5 years on the $x$-axis and 2 cm to represent $\$ 50$ on the $y$-axis, draw an $x$-axis for $0 \leqslant x \leqslant 40$ and a $y$-axis for $0 \leqslant y \leqslant 500$.

Plot the five points in the table and draw a smooth curve through'them.
(d) Use your graph to estimate
(i) how many dollars Alaric will have after 25 years,
(ii) how many years, to the nearest year, it takes for Alaric to have $\$ 200$.
(e) Beatrice invests $\$ 100$ at $7 \%$ per year simple interest.
(i) Show that after 20 years Beatrice has $\$ 240$.
(ii) How many dollars will Beatrice have after 40 years?
(iii) On the same grid, draw a graph to show how the $\$ 100$ which Beatrice invests will increase during the 40 years.
(f) Alaric first has more than Beatrice after $n$ years.

Use your graphs to find the value of $n$.

1 Marcus receives $\$ 800$ from his grandmother.
(a) He decides to spend $\$ 150$ and to divide the remaining $\$ 650$ in the ratio savings $:$ holiday $=9: 4$.

Calculate the amount of his savings.

## Answer(a) \$

(b) (i) He uses $80 \%$ of the $\$ 150$ to buy some clothes.

Calculate the cost of the clothes.

> Answer(b)(i) \$
(ii) The money remaining from the $\$ 150$ is $37 \frac{1}{2} \%$ of the cost of a day trip to Cairo. Calculate the cost of the trip.

> Answer(b)(ii) \$
(c) (i) Marcus invests $\$ 400$ of his savings for 2 years at $5 \%$ per year compound interest. Calculate the amount he has at the end of the 2 years.
Answer(c)(i) \$
(ii) Marcus's sister also invests $\$ 400$, at $r \%$ per year simple interest.

At the end of 2 years she has exactly the same amount as Marcus.
Calculate the value of $r$.

$$
\text { Answer(c)(ii) } r=
$$

1 Write the numbers in order of size with the smallest first.

| $\sqrt{10}$ | 3.14 | $\frac{22}{7}$ | $\pi$ |
| :--- | :--- | :--- | :--- |



2 Michel changed $\$ 600$ into pounds $(£)$ when the exchange rate was $£ 1=\$ 2.40$.
He later changed all the pounds back into dollars when the exchange rate was $£ 1=\$ 2.60$.
How many dollars did he receive?
$3 \quad p$ is the largest prime number between 50 and 100 .
$q$ is the smallest prime number between 50 and 100 .
Calculate the value of $p-q$.

> Answer

4 A person in a car, travelling at 108 kilometres per hour, takes 1 second to go past a building on the side of the road.

Calculate the length of the building in metres.

5 Calculate the value of $5\left(6 \times 10^{3}+400\right)$, giving your answer in standard form.

6 Calculate the value of $\frac{1}{2} \sqrt{\frac{1}{2}+\frac{1}{2} \sqrt{\frac{1}{2}}}$


The top of a desk is made from a rectangle and a quarter circle.
The rectangle measures 0.8 m by 1.4 m .
Calculate the surface area of the top of the desk.

9 A cyclist left Melbourne on Wednesday 21 May at 0945 to travel to Sydney. The journey took 97 hours.

Write down the day, date and time that the cyclist arrived in Sydney.
Answer Day ................................. Date ..................... Time ı............... [3]

10


NOTTO
1.5 m

SCALE

The diagram represents a rectangular gate measuring 1.5 m by 3.5 m .
It is made from eight lengths of wood.

Calculate the total length of wood needed to make the gate.

1 During one week in April, in Quebec, the daily minimum temperatures were
$-5^{\circ} \mathrm{C}$,
$-1^{\circ} \mathrm{C}$,
$3^{\circ} \mathrm{C}$,
$2^{\circ} \mathrm{C}$,
$-2^{\circ} \mathrm{C}$,
$0^{\circ} \mathrm{C}$,
$6^{\circ} \mathrm{C}$.

Write down
(a) the lowest of these temperatures,

Answer(a)
${ }^{\circ} \mathrm{C} \quad[1]$
(b) the range of these temperatures.

Answer(b)
${ }^{\circ} \mathrm{C}$

2
$\sqrt{23}$
48\%
4.80 $\frac{53}{11}$

Write the numbers in order of size with the largest first.

3 Ricardo changed $\$ 600$ into pounds ( $£$ ) when the exchange rate was $\$ 1=£ 0.60$.
He later changed all the pounds back into dollars when the exchange rate was $\$ 1=£ 0.72$.
How many dollars did he receive?

> Answer \$

4 The maximum speed of a car is $252 \mathrm{~km} / \mathrm{h}$.
Change this speed into metres per second.

> Answer
$\mathrm{m} / \mathrm{s}$

5 Amalie makes a profit of $20 \%$ when she sells a shirt for $\$ 21.60$.
Calculate how much Amalie paid for the shirt.
$6 \quad 3^{x} \times 9^{4}=3^{n}$.
Find $n$ in terms of $x$.


8 Write as a single fraction in its simplest form

$$
\frac{x}{3}+\frac{x-1}{2} .
$$

91 second $=10^{6}$ microseconds.
Change $3 \times 10^{13}$ microseconds into minutes. Give your answer in standard form.

10 The length of each side of an equilateral triangle is 74 mm , correct to the nearest millimetre.
Calculate the smallest possible perimeter of the triangle.


1 A school has 220 boys and 280 girls.
(a) Find the ratio of boys to girls, in its simplest form.

> Answer(a)
:
(b) The ratio of students to teachers is $10: 1$.

Find the number of teachers.

> Answer(b)
(c) There are 21 students on the school's committee.

The ratio of boys to girls is $3: 4$.
Find the number of girls on the committee.

Answer(c)
[2]
(d) The committee organises a disco and sells tickets.
$35 \%$ of the school's students each buy a ticket. Each ticket costs \$1.60.
Calculate the total amount received from selling the tickets.

(e) The cost of running the disco is $\$ 264$.

This is an increase of $10 \%$ on the cost of running last year's disco.
Calculate the cost of running last year's disco.

1 Alberto and Maria share $\$ 240$ in the ratio $3: 5$.
(a) Show that Alberto receives $\$ 90$ and Maria receives $\$ 150$.

Answer(a)
(b) (i) Alberto invests his $\$ 90$ for 2 years at $r \%$ per year simple interest.

At the end of 2 years the amount of money he has is $\$ 99$.
Calculate the value of $r$.

$$
\text { Answer(b)(i) } r=
$$

(ii) The $\$ 99$ is $60 \%$ of the cost of a holiday. Calculate the cost of the holiday.

Answer(b)(ii) \$
(c) Maria invests her $\$ 150$ for 2 years at $4 \%$ per year compound intêrest.

Calculate the exact amount Maria has at the end of 2 years.

(d) Maria continues to invest her money at $4 \%$ per year compound interest.

After 20 years she has $\$ 328.67$.
(i) Calculate exactly how much more this is than $\$ 150$ invested for 20 years at $4 \%$ per year simple interest.
Answer(d)(i) \$
(ii) Calculate $\$ 328.67$ as a percentage of $\$ 150$.

1 Daniella is 8 years old and Edward is 12 years old.
(a) Their parents give them some money in the ratio of their ages.
(i) Write the ratio Daniella's age : Edward's age in its simplest form.
Answer(a)(i) ............. : ............. [1]
(ii) Daniella receives $\$ 30$.

Show that Edward receives $\$ 45$.
Answer(a)(ii)
(iii) What percentage of the total amount of money given by their parents does Edward receive?
(b) Daniella invests her $\$ 30$ at $3 \%$ per year, compound interest.

Calculate the amount Daniella has after 2 years.
Give your answer correct to 2 decimal places.

Answer(b) \$
[3]
(c) Edward also invests $\$ 30$.

He invests this money at a rate of $r \%$ per year, simple interest.
After 5 years he has a total amount of $\$ 32.25$.
Calculate the value of $r$.

1 A concert hall has 1540 seats.
Calculate the number of people in the hall when $55 \%$ of the seats are occupied.

## Answer

2 (a) Write down in figures the number twenty thousand three hundred and seventy six.


5 Mark and Naomi share $\$ 600$ in the ratio $\quad$ Mark : Naomi $=5: 1$.
Calculate how much money Naomi receives.

6 Calculate the area of a circle with radius 6.28 centimetres.
$\square$
Answer

7 The scale on a map is $1: 20000$.
Calculate the actual distance between two points which are 2.7 cm apart on the map.
Give your answer in kilometres.


> Answer
km [2]

8 (a) Find $m$ when $4^{m} \times 4^{2}=4^{12}$.

> Answer(a) m=
(b) Find $p$ when $6^{p} \div 6^{7}=6^{2}$.

$$
\operatorname{Answer}(b) p=
$$

12 (a) Write 1738.279 correct to 1 decimal place.

> Answer(a)
(b) Write 28700 in standard form.

> Answer(b)
(c) The mass of a ten-pin bowling ball is 7 kg to the nearest kilogram.

Write down the lower bound of the mass of the ball.

> Answer(c)

13 Paulo invests \$3000 at a rate of $4 \%$ per year compound interest.
Calculate the total amount Paulo has after 2 years.
Give your answer correct to the nearest dollar.

> Answer(a)
(b) The distance from Barcelona to Paris is 827 km .

Calculate the average speed of the train in kilometres per hour.

15 (a) The table shows part of a railway timetable.

| Peartree <br> Station | arrival time | 1258 | 1356 | 1454 | 1552 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | departure time | 1307 | 1405 | 1503 | 1601 |

(i) Each train waits the same number of minutes at Peartree Station.

Write down how many minutes each train waits.

> Answer(a)(i)
$\qquad$
(ii) Janine is at Peartree Station at 3 pm .

At what time does the next train depart?
(b) The average temperature each month in Moscow and Helsinki is recorded. The table shows this information from January to June.

|  | January | February | March | April | May | June |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature in <br> Moscow $\left({ }^{\circ} \mathrm{C}\right)$ | -16 | -14 | -8 | 1 | 8 | 11 |
| Temperature in <br> Helsinki $\left({ }^{\circ} \mathrm{C}\right)$ | -9 | -10 | -7 | -1 | 4 | 10 |

(i) Find the difference in temperature between Moscow and Helsinki in January.
Answer(b)(i)
(ii) Find the increase in temperature in Helsinki from March to June.

5 Show that

$$
1 \frac{5}{9} \div 1 \frac{7}{9}=\frac{7}{8} .
$$

Write down all the steps in your working.
Answer

6

Which of the following could be a value of $p$ ?

Answer

7 Calculate $324 \times 17$.
Give your answer in standard form, correct to 3 significant figures.

13 (a) Rewrite this calculation with all the numbers rounded to 1 significant figure.

$$
\frac{77.8}{21.9-3.8 \times 4.3}
$$

## Answer(a)

(b) Use your answer to part (a) to work out an estimate for the calculation.
Answer(b)
(c) Use your calculator to find the actual answer to the calculation in part (a).

Give your answer correct to 1 decimal place.


14 (a) Complete the list to show all the factors of 18.
1 ,
2,

......... , $\qquad$ ,

$$
18
$$

(b) Write down the prime factors of 18 .

> Answer(b)
(c) Write down all the multiples of 18 between 50 and 100 .

10


In triangle $A B C, A B=12 \mathrm{~cm}$, angle $C=90^{\circ}$ and angle $A=27^{\circ}$.
Calculate the length of $A C$.

11


In the rectangle $A B C D, A B=9 \mathrm{~cm}$ and $B D=12 \mathrm{~cm}$.
Calculate the length of the side $B C$.

Answer $B C=$
cm [3]

12 (a) Write 16460000 in standard form.
Answer(a)
(b) Calculate $7.85 \div\left(2.366 \times 10^{2}\right)$, giving your answer in standard form.

13 (a) Find the value of $x$ when $\frac{18}{24}=\frac{27}{x}$.

$$
\text { Answer(a) } x=
$$

(b) Show that $\frac{2}{3} \div 1 \frac{1}{6}=\frac{4}{7}$.

Write down all the steps in your working.
Answer(b)

14 (a) A drinking glass contains 55 cl of water. Write 55 cl in litres.
(b) The mass of grain in a sack is 35 kg . The grain is divided equally into 140 bags.

Calculate the mass of grain in each bag.
Give your answer in grams.

Answer(b)
g [2]

15 (a) Write 67.499 correct to the nearest integer.

Answer(a)
(b) Write 0.003040506 correct to 3 significant figures.

Answer(b)
(c) $d=56.4$, correct to 1 decimal place.

Write down the lower bound of $d$.

Answer (c)
[1]

18 Eva invests $\$ 120$ at a rate of $3 \%$ per year compound interest.
Calculate the total amount Eva has after 2 years. Give your answer correct to 2 decimal places.

## Answer \$

19 At a ski resort the temperature, in ${ }^{\circ} \mathrm{C}$, was measured every 4 hours during one day. The results were $-12^{\circ}, \quad-13^{\circ}, \quad-10^{\circ}, \quad 4^{\circ}, \quad 4^{\circ}, \quad-6^{\circ}$.
(a) Find the difference between the highest and the lowest of these temperatures.
(b) Find
(i) the mean,

(ii) the median,

Answer(b)(ii) $\qquad$
(iii) the mode.

1 A concert hall has 1540 seats.
Calculate the number of people in the hall when $55 \%$ of the seats are occupied.

3 Calculate $81^{0.25} \div 4^{-2}$.


4 (a) Find $m$ when $4^{m} \times 4^{2}=4^{12}$.

$$
\text { Answer(a) } m=
$$

(b) Find $p$ when $6^{p} \div 6^{5}=\sqrt{6}$.

5 A hummingbird beats its wings 24 times per second.
(a) Calculate the number of times the hummingbird beats its wings in one hour.
Answer(a)
(b) Write your answer to part (a) in standard form.
Answer(b)

6


A company makes solid chocolate eggs and their shapes are mathematicallysimilar.
The diagram shows eggs of height 2 cm and 6 cm .
The mass of the small egg is 4 g .
Calculate the mass of the large egg.

> Answer

11 A rectangular photograph measures 23.3 cm by 19.7 cm , each correct to 1 decimal place. Calculate the lower bound for
(a) the perimeter,

> Answer(a)
cm [2]
(b) the area.


12 A train leaves Barcelona at 2128 and takes 10 hours and 33 minutes to reach Paris.
(a) Calculate the time the next day when the train arrives in Paris.

> Answer(a)
(b) The distance from Barcelona to Paris is 827 km .

Calculate the average speed of the train in kilometres per hour.

13 The scale on a map is 1:20000.
(a) Calculate the actual distance between two points which are 2.7 cm apart on the map. Give your answer in kilometres.

Answer(a)
km [2]
(b) A field has an area of $64400 \mathrm{~m}^{2}$.

Calculate the area of the field on the map in $\mathrm{cm}^{2}$.


1 In the right-angled triangle $A B C, \cos C=\frac{4}{5}$. Find angle $A$.


2 Which of the following numbers are irrational?

$$
\begin{array}{lll}
\frac{2}{3} & \sqrt{36} & \sqrt{3}+\sqrt{6}
\end{array}
$$

3 Show that

Write down all the steps in your working.
Answer

$$
\frac{3}{5}<p<\frac{2}{3}
$$

Which of the following could be a value of $p$ ?
$\frac{16}{27} \quad 0.67 \quad 60 \% \quad(0.8)^{2} \quad \sqrt{\frac{4}{9}}$

5 A meal on a boat costs 6 euros ( $€$ ) or 11.5 Brunei dollars (\$).
In which currency does the meal cost less, on a day when the exchange rate is $€ 1=\$ 1.9037$ ? Write down all the steps in your working.

> Answer

6 Use your calculator to find the value of $2^{\sqrt{3}}$.
Give your answer correct to 4 significant figures.

> Answer

7 Solve the equation $4 x+6 \times 10^{3}=8 \times 10^{4}$.
Give your answer in standard form.
$8 \quad p$ varies directly as the square root of $q$.
$p=8$ when $q=25$.
Find $p$ when $q=100$.

9 Ashraf takes 1500 steps to walk $d$ metres from his home to the station.
Each step is 90 centimetres correct to the nearest 10 cm .
Find the lower bound and the upper bound for $d$.

10 The table shows the opening and closing times of a café.

|  | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Opening time | 0600 | 0600 | 0600 | 0600 | 0600 | $(a)$ | 0800 |
| Closing time | 2200 | 2200 | 2200 | 2200 | 2200 | 2200 | 1300 |

(a) The café is open for a total of 100 hours each week.

Work out the opening time on Saturday.
(b) The owner decides to close the café at a later time on Sunday. This increases the total number of hours the café is open by $4 \%$.
Work out the new closing time on Sunday.

11 Rearrange the formula $c=\frac{4}{a-b}$ to make $a$ the subject.

$$
\text { Answer } a=
$$

4 Helen measures a rectangular sheet of paper as 197 mm by 210 mm , each correct to the nearest millimetre.
Calculate the upper bound for the perimeter of the sheet of paper.

5

The sketch shows the graph of $y=a x^{n}$ where $a$ and $n$ are integers.
Write down a possible value for $a$ and a possible value for $n$.


6 (a) Write 16460000 in standard form.
Answer (a)
(b) Calculate $7.85 \div\left(2.366 \times 10^{2}\right)$, giving your answer in standard form.

7 (a) Find the value of $x$ when $\frac{18}{24}=\frac{27}{x}$.

$$
\text { Answer (a) } x=
$$

(b) Show that $\frac{2}{3} \div 1 \frac{1}{6}=\frac{4}{7}$.

Write down all the steps in your working.
Answer(b)


9 Eva invests \$120 at a rate of 3\% per year compound interest.
Calculate the total amount Eva has after 2 years.
Give your answer correct to 2 decimal places.

12 Federico changed 400 euros ( $€$ ) into New Zealand dollars (NZ\$) at a rate of $€ 1=\mathrm{NZ}$ 2.1. He spent $x$ New Zealand dollars and changed the rest back into euros at a rate of $€ 1=\mathrm{NZ} \$ d$.

Find an expression, in terms of $x$ and $d$, for the number of euros Federico received.


18 Simplify the following.
(a) $\left(3 x^{3}\right)^{3}$

> Answer(a)
(b) $\left(125 x^{6}\right)^{\frac{2}{3}}$

19 The scale of a map is $1: 250000$.
(a) The actual distance between two cities is 80 km .

Calculate this distance on the map. Give your answer in centimetres.
(b) On the map a large forest has an area of $6 \mathrm{~cm}^{2}$.

Calculate the actual area of the forest. Give your answer in square kilometres.

1 Mr and Mrs Clark and their three children live in the USA and take a holiday in Europe.
(a) Mr Clark changes $\$ 500$ into euros $(€)$ when the exchange rate is $€ 1=\$ 1.4593$.

Calculate how much he receives.
Give your answer correct to 2 decimal places.

## Answer(a) €

[2]
(b) Tickets for an amusement park cost $€ 62$ for an adult and $€ 52$ for a child.

Work out the cost for Mr and Mrs Clark and their three children to visit the park.
(c) Mr Clark sees a notice:
[3]

Work out $€ 200$ as a percentage of your answer to part (b).
(d) Mrs Clark buys 6 postcards at $€ 0.98$ each. She pays with a $€ 10$ note.

Calculate how much change she will receive.

$$
\text { Answer }(d) €
$$

[2]
(e) Children under a height of 130 cm are not allowed on one of the rides in the park.

Helen Clark is 50 inches tall.

Use 1 inch $=2.54 \mathrm{~cm}$ to show that she will not be allowed on this ride .

Answer(e)


6 (a) 103

112
125
132
144
159
161
From the list above, write down
(i) a square number,

> Answer(a)(i)
(ii) a cube number,

Answer(a)(ii)
(iii) a prime number,

> Answer(a)(iii)
(iv) an odd number which is a multiple of 3 .

> Answer(a)(iv)
(b) Write 88 as a product of prime numbers.

## Answer(b)

(c) Find the highest common factor of 72 and 96 .


> Answer(c)
(d) Find the lowest common multiple of 15 and 20.

1 Falla buys 3000 square metres of land for a house and garden.
The garden is divided into areas for flowers, vegetables and grass.
He divides the land in the following ratio.
house : flowers : vegetables : grass $=4: 7: 8: 5$
(a) (i) Show that the area of land used for flowers is $875 \mathrm{~m}^{2}$.

Answer(a)(i)
(ii) Calculate the area of land used for the house.
(b) Write down the fraction of land used for vegetables. Give your answer in its simplest form.
(c) During the first year Falla plants flowers in $64 \%$ of the $875 \mathrm{~m}^{2}$.

Calculate the area he plants with flowers.
Answer(c)
(d) Falla sells some of the vegetables he grows.

These vegetables cost $\$ 85$ to grow.
He sells them for $\$ 105$.
Calculate his percentage profit.

\% [3]
(e) To buy the land Falla borrowed $\$ 5000$ at a rate of $6.4 \%$ compound interest for 2 years.

Calculate the total amount he pays back at the end of the 2 years.
Give your answer correct to the nearest dollar.

1 At a theatre, adult tickets cost $\$ 5$ each and child tickets cost $\$ 3$ each.
(a) Find the total cost of 110 adult tickets and 85 child tickets.
Answer(a) \$
(b) The total cost of some tickets is $\$ 750$.

There are 120 adult tickets.

Work out the number of child tickets.

## Answer(b)

(c) The ratio of the number of adults to the number of children during one performance is adults $:$ children $=3: 2$.
(i) The total number of adults and children in the theatre is 150 .

Find the number of adults in the theatre.
(ii) For this performance, find the ratio total cost of adult tickets : total cost of child tickets. Give your answer in its simplest form.

Answer(c)(ii) $\qquad$ :
(d) The $\$ 5$ cost of an adult ticket is increased by $30 \%$.

Calculate the new cost of an adult ticket.

> Answer(d) \$
(e) The cost of a child ticket is reduced from $\$ 3$ to $\$ 2.70$.

Calculate the percentage decrease in the cost of a child ticket.

1 A school has a sponsored swim in summer and a sponsored walk in winter. In 2010, the school raised a total of $\$ 1380$.
The ratio of the money raised in $\quad$ summer: winter $=62: 53$.
(a) (i) Show clearly that $\$ 744$ was raised by the swim in summer.

Answer (a)(i)
(ii) Alesha’s swim raised $\$ 54.10$. Write this as a percentage of $\$ 744$.

> Answer(a)(ii)
\% [1]
(iii) Bryan's swim raised $\$ 31.50$.

He received 75 cents for each length of the pool which he swam.
Calculate the number of lengths Bryan swam.
(b)

> Answer(a)(iiil)

1 (a) Work out the following.
(i) $\frac{1}{0.2^{2}}$

> Answer(a)(i)
(ii) $\sqrt{5.1^{2}+4 \times 7.3^{2}}$

Answer(a)(ii)
(iii) $25^{\frac{1}{2}} \times 1000^{-\frac{2}{3}}$

Answer(a)(iii)
[2]
(b) Mia invests $\$ 7500$ at $3.5 \%$ per year simple interest.

Calculate the total amount she has after 5 years.
(c) Written as the product of prime factors $48=2^{4} \times 3$.
(i) Write 60 as the product of prime factors.

Answer(c)(i)
(ii) Work out the highest common factor (HCF) of 48 and 60.

## Answer(c)(ii)

[2]
(iii) Work out the lowest common multiple (LCM) of 48 and 60.

1 Lucy works in a clothes shop.
(a) In one week she earned $\$ 277.20$.
(i) She spent $\frac{1}{8}$ of this on food.

Calculate how much she spent on food.

> Answer(a)(i) \$
(ii) She paid $15 \%$ of the $\$ 277.20$ in taxes.

Calculate how much she paid in taxes.

Answer(a)(ii) \$
(iii) The $\$ 277.20$ was $5 \%$ more than Lucy earned in the preyious week. Calculate how much Lucy earned in the previous week.

## Answer(a)(iii)\$

(b) The shop sells clothes for men, women and children.
(i) In one day Lucy sold clothes with a total value of $\$ 2200$ in the ratio

$$
\text { men }: \text { women }: \text { children }=2: 5: 4 .
$$

Calculate the value of the women's clothes she sold.

> Answer(b)(i) \$
(ii) The $\$ 2200$ was $\frac{44}{73}$ of the total value of the clothes sold in the shop on this day. Calculate the total value of the clothes sold in the shop on this day.

1 The table shows the maximum daily temperatures during one week in Punta Arenas.

| Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Sunday |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2^{\circ} \mathrm{C}$ | $3^{\circ} \mathrm{C}$ | $1^{\circ} \mathrm{C}$ | $2.5^{\circ} \mathrm{C}$ | $-1.5^{\circ} \mathrm{C}$ | $1^{\circ} \mathrm{C}$ | $2^{\circ} \mathrm{C}$ |

(a) By how many degrees did the maximum temperature change between Thursday and Friday?
Answer (a)
$\qquad$
(b) What is the difference between the greatest and the least of these temperatures?

Answer (b)

2 Nyali paid \$62 for a bicycle. She sold it later for \$46.
What was her percentage loss?

Answer

3 Three sets $A, B$ and $K$ are such that $A \subset K, B \subset K$ and $A \cap B=\varnothing$.
Draw a Venn diagram to show this information.


4 Alejandro goes to Europe for a holiday.
He changes 500 pesos into euros at an exchange rate of 1 euro $=0.975$ pesos.
How much does he receive in euros? Give your answer correct to 2 decimal places.

Answer
euros

5 Write the four values in order, smallest first.

$$
\frac{1}{1000}, \quad \frac{11}{1000}, \quad 0.11 \%, \quad 0.0108
$$

Answer $<$ $\qquad$ $<$ $\qquad$ $<$

7 Find the exact value of
(a) $3^{-2}$,

(b) $\left(1 \frac{7}{9}\right)^{\frac{1}{2}}$.

Answer (b)

8 The length of a road is 380 m , correct to the nearest 10 m .
Maria runs along this road at an average speed of $3.9 \mathrm{~m} / \mathrm{s}$.
This speed is correct to 1 decimal place.
Calculate the greatest possible time taken by Maria.

1 (a) At an athletics meeting, Ben's time for the 10000 metres race was 33 minutes exactly and he finished at 1517 .
(i) At what time did the race start?
(ii) What was Ben's average speed for the race? Give your answer in kilometres per hour.
(iii) The winner finished 51.2 seconds ahead of Ben. How long did the winner take to run the 10000 metres?
(b) The winning distance in the javelin competition was 80 metres.

Otto's throw was $95 \%$ of the winning distance.
Calculate the distance of Otto's throw.
(c) Pamela won the long jump competition with a jump of 6.16 metres.

This was $10 \%$ further than Mona's jump.
How far did Mona jump?


1 Work out $\frac{2+12}{4+3 \times 8}$.

Answer

2 The altitude of Death Valley is -86 metres.
The altitude of Mount Whitney is 4418 metres.
Calculate the difference between these two altitudes.

Answer
m [1]

3 The first five terms of a sequence are $4,9,16,25,36, \ldots$
Find
(a) the 10th term,
Answer (a).
(b) the $n$th term.

4 Rearrange the quantities in order with the smallest first.

Answer $\qquad$ .. $<$. $<$.
$5 \mathscr{E}=\left\{-2 \frac{1}{2},-1, \sqrt{2}, 3.5, \sqrt{30}, \sqrt{36}\right\}$
$X=\{$ integers $\}$
$Y=\{$ irrational numbers $\}$
List the members of
(a) $X$,

$$
\begin{equation*}
\text { Answer (a) } X=\{ \tag{1}
\end{equation*}
$$

(b) $Y$.

$$
\text { Answer (b) } Y=\{.
$$

6 Abdul invested $\$ 240$ when the rate of simple interest was $r \%$ per year.
After $m$ months the interest was $\$ I$.
Write down and simplify an expression for $I$, in terms of $m$ and $r$.

$$
\text { Answer } I=
$$

7 A baby was born with a mass of 3.6 kg .
After three months this mass had increased to 6 kg .
Calculate the percentage increase in the mass of the baby.

8 (a) $3^{x}=\frac{1}{3}$.
Write down the value of $x$.
(b) $5^{y}=k$.

Find $5^{y+1}$, in terms of $k$.

Answer (b) $5^{y+1}=$

9 (a) 32493 people were at a football match.
Write this number to the nearest thousand.

Answer (a).
(b) At another match there were 25500 people, to the nearest hundred.

Complete the inequality about $n$, the number of people at this match.

Answer (b)
$\leqslant n<$

10 When cars go round a bend there is a force, $F$, between the tyres and the ground.
$F$ varies directly as the square of the speed, $v$.
When $v=40, F=18$.
Find $F$ when $v=32$.

Answer $F=$

11 In April 2001, a bank gave the following exchange rates.
1 euro $=0.623$ British pounds.
1 euro = 1936 Italian lire .
(a) Calculate how much one pound was worth in lire.

## Answer (a)

(b) Calculate how much one million lire was worth in pounds.

## Answer (b)

pounds [1

12 The diagram shows the graphs of $y=\sin x^{\circ}$ and $y=\cos x^{\circ}$.


Find the values of $x$ between 0 and 360 for which
(a) $\sin x^{\circ}=\cos x^{\circ}$,

Answer (a) $x=$. $\qquad$ or $x=$
(b) $\sin x^{\circ}=\sin 22.5^{\circ}(x \neq 22.5)$.

18 The population of Europe is 580000000 people.
The land area of Europe is 5900000 square kilometres.
(a) Write 580000000 in standard form.

> Answer (a).
(b) Calculate the number of people per square kilometre, to the nearest whole number.

Answer (b)
(c) Calculate the number of square metres per person.
$\mathrm{m}^{2}$ [2]

1 A train starts its journey with 240 passengers.
144 of the passengers are adults and the rest are children.
(a) Write the ratio Adults: Children in its lowest terms.
(b) At the first stop, $37 \frac{1}{2} \%$ of the adults and $\frac{1}{3}$ of the children get off the train. 20 adults and $x$ children get onto the train.
The total number of passengers on the train is now 200.
(i) How many children got off the train?
(ii) How many adults got off the train?
(iii) How many adult passengers are on the train as it sets off again?
(iv) What is the value of $x$ ?
(c) After a second stop, there are 300 passengers on the train and the ratio

Men:Women: Children is 6:5:4.
Calculate the number of children now on the train.
(d) On Tuesday the train journey took 7 hours and 20 minutes and began at 1353 .
(i) At what time did the train journey end?
(ii) Tuesday's time of 7 hours 20 minutes was $10 \%$ more than Monday ${ }^{2}$ journey time. How many minutes longer was Tuesday's journey?

1 A pattern of numbers is shown below.


Write down the value of $x$.

2 Calculate $(3+3 \sqrt{3})^{3}$ giving your answer correct to 1 decimal place.

3 From the list of numbers $\frac{22}{7}, \pi, \sqrt{14}, \sqrt{16}, 27.4, \frac{65}{13}$ write down
(a) one integer,
(b) one irrational number


7 The air resistance $(R)$ to a car is proportional to the square of its speed $(v)$. When $R=1800, v=30$.
Calculate $R$ when $v=40$.

$$
\text { Answer } R=
$$

8 In 1997 the population of China was $1.24 \times 10^{9}$.
In 2002 the population of China was $1.28 \times 10^{9}$.
Calculate the percentage increase from 1997 to 2002.


1 The population of Newtown is 45000. The population of Villeneuve is 39000 .
(a) Calculate the ratio of these populations in its simplest form.
(b) In Newtown, 28\% of the population are below the age of twenty.

Calculate how many people in Newtown are below the age of twenty.
(c) In Villeneuve, 16000 people are below the age of twenty.

Calculate the percentage of people in Villeneuve below the age of twenty.
(d) The population of Newtown is $125 \%$ greater than it was fifty years ago. Calculate the population of Newtown fifty years ago.
(e) The two towns are combined and made into one city called Monocity. In Monocity the ratio of
men : women :children is $12: 13: 5$.
Calculate the number of children in Monocity.


The number of tennis balls $(T)$ in the diagram is given by the formula

$$
T=\frac{1}{2} n(n+1),
$$

where $n$ is the number of rows.
The diagram above has 4 rows.
How many tennis balls will there be in a diagram with 20 rows?

## Answer

2 Calculate the value of $2\left(\sin 15^{\circ}\right)\left(\cos 15^{\circ}\right)$.

## Answer



4 Write down the next term in each of the following sequences.
(a) $8.2, \quad 6.2, \quad 4.2, \quad 2.2, \quad 0.2, \ldots$

Answer(a)
(b) $1,3, \quad 6,10,15, \ldots$

Answer(b)

5 Celine invests $\$ 800$ for 5 months at $3 \%$ simple interest per year.
Calculate the interest she receives.

$$
(0.8)^{\frac{1}{2}}, \quad 0.8, \quad \sqrt{0.8}, \quad(0.8)^{-1}, \quad(0.8)^{2}
$$

From the numbers above, write down
(a) the smallest,
Answer(a) ...................................................... [1]
(b) the largest.
Answer(b) ..... [1]
$7 \quad \mathrm{f}(x)=10^{x}$.
(a) Calculate $\mathrm{f}(0.5)$.
(b) Write down the value of $\mathrm{f}^{-1}(1)$.

9 Write the number 2381.597 correct to
(a) 3 significant figures,
Answer(a) ........................................................ [1]
(b) 2 decimal places,

Answer(b)
(c) the nearest hundred.

Answer(c)

10 The mass of the Earth is $\frac{1}{95}$ of the mass of the planet Saturn.
The mass of the Earth is $5.97 \times 10^{24}$ kilograms.
Calculate the mass of the planet Saturn, giving your answer in standard form, correct to 2 significant figures.

Answer

11 A large conference table is made from four rectangular sections and four corner sections.
Each rectangular section is 4 m long and 1.2 m wide.
Each corner section is a quarter circle, radius 1.2 m .


Each person sitting at the conference table requires one metre of its outside perimeter.
Calculate the greatest number of people who can sit around the outside of the table.
Show all your working.

1 A Spanish family went to Scotland for a holiday.
(a) The family bought 800 pounds $(\mathfrak{£})$ at a rate of $\mathfrak{£} 1=1.52$ euros $(€)$.

How much did this cost in euros?
(b) The family returned home with $£ 118$ and changed this back into euros.

They received $€ 173.46$.
Calculate how many euros they received for each pound.
(c) A toy which costs $€ 11.50$ in Spain costs only $€ 9.75$ in Scotland.

Calculate, as a percentage of the cost in Spain, how much less it costs in Scotland.
(d) The total cost of the holiday was $€ 4347.00$.

In the family there were 2 adults and 3 children.
The cost for one adult was double the cost for one child.
Calculate the cost for one child.
(e) The original cost of the holiday was reduced by $10 \%$ to $€ 4347.00$.

Calculate the original cost.
(f) The plane took 3 hours 15 minutes to return to Spain. The length of this journey was 2350 km .
Calculate the average speed of the plane in
(i) kilometres per hour,
(ii) metres per second.

1 Two quantities $c$ and $d$ are connected by the formula $c=2 d+30$.
Find $c$ when $d=-100$

2
(a)

$$
\frac{2}{3}+\frac{5}{6}=\frac{x}{2} .
$$

Find the value of $x$.

Answer(a) $x=$
(b)

$$
\frac{5}{3} \div \frac{3}{y}=\frac{40}{9}
$$

Find the value of $y$.

3 Use your calculator to work out
(a) $\sqrt{ }\left(7+6 \times 243^{0.2}\right)$,

> Answer(a)
(b) $2-\tan 30^{\circ} \times \tan 60^{\circ}$.

4 Angharad sleeps for 8 hours each night, correct to the nearest 10 minutes.
The total time she sleeps in the month of November ( 30 nights) is $T$ hours.
Between what limits does $T$ lie?

Answer
$\leqslant T<$

5


7 Find the value of $n$ in each of the following statements.
(a) $32^{n}=1$

$$
\begin{equation*}
\text { Answer(a) } n= \tag{1}
\end{equation*}
$$

(b) $32^{n}=2$

$$
\text { Answer(b) } n=
$$

(c) $32^{n}=8$

$$
\text { Answer(c) } n=
$$

8 The Canadian Maple Leaf train timetable from Toronto to Buffalo is shown below.

| Toronto | 1030 |
| :--- | :---: |
| Oakville | 1052 |
| Aldershot | 1107 |
| Grimsby | 1141 |
| St Catharines | 1159 |
| Niagra Falls | 1224 |
| Buffalo | 1325 |

(a) How long does the journey take from Toronto to Buffalo?

Answer (a)
$\min [1]$
(b) This journey is 154 kilometres. Calculate the average speed of the train.

9 For each of the following sequences, write down the next term.
(a) $2,3,5,8,13, \ldots$

> Answer(a)
(b) $x^{6}, 6 x^{5}, 30 x^{4}, 120 x^{3}, \ldots$

> Answer(b)
(c) $2,6,18,54,162, \ldots$

1 Maria, Carolina and Pedro receive $\$ 800$ from their grandmother in the ratio
Maria: Carolina: Pedro = 7:5:4.
(a) Calculate how much money each receives.
(b) Maria spends $\frac{2}{7}$ of her money and then invests the rest for two years at 5\% per year simple interest.
How much money does Maria have at the end of the two years?
(c) Carolina spends all of her money on a hi-fi set and two years later sells it at a loss of $20 \%$. How much money does Carolina have at the end of the two years?
(d) Pedro spends some of his money and at the end of the two years he has $\$ 100$.

Write down and simplify the ratio of the amounts of money Maria, Carolina and Pedro have at the end of the two years.
(e) Pedro invests his $\$ 100$ for two years at a rate of $5 \%$ per year compound interest.

Calculate how much money he has at the end of these two years.

1 Use a calculator to find the value of

$$
\sqrt{(5.4(5.4-4.8)(5.4-3.4)(5.4-2.6))} .
$$

(a) Write down all the figures in your calculator display.
Answer(a)
(b) Give your answer correct to 1 decimal place.

2 Use the formula

$$
P=\frac{V^{2}}{R}
$$

to calculate the value of $P$ when $V=6 \times 10^{6}$ and $R=7.2 \times 10^{8}$.

(a) the order of rotational symmetry,
Answer(a)
(b) the number of lines of symmetry.

4 When $0<x<0.9$, write the following in order of size with the smallest first.
$\cos x^{\circ}$
$x^{2}$
$x^{-1}$

Answer $\qquad$ $<$ $\qquad$ ................. $<$
$5 \quad \frac{4 c}{5}-\frac{3 c}{35}=\frac{10}{7}$. Find $c$.

6

$$
p=\frac{0.002751 \times 3400}{(9.8923+24.7777)^{2}} .
$$

(a) In the spaces provided, write each number in this calculation correct to 1 significant figure.

(b) Use your answer to part (a) to estimate the value of $p$.

8 (a) In October the cost of a car in euros was $€ 20000$. The cost of this car in pounds was $£ 14020$.
Calculate the exact value of the exchange rate in October.

$$
\text { Answer }(a) € 1=£
$$

(b) In November the car still cost $€ 20000$ and the exchange rate was $€ 1=£ 0.6915$. Calculate the difference, in pounds, between the cost in October and November.


1 Each year a school organises a concert.
(a) (i) In 2004 the cost of organising the concert was $\$ 385$.

In 2005 the cost was $10 \%$ less than in 2004.

Calculate the cost in 2005.
(ii) The cost of \$385 in 2004 was $10 \%$ more than the cost in 2003 . Calculate the cost in 2003.
(b) (i) In 2006 the number of tickets sold was 210 .

The ratio
Number of adult tickets : Number of student tickets was 23:19.

How many adult tickets were sold?
(ii) Adult tickets were $\$ 2.50$ each and student tickets were $\$ 1.50$ each.

Calculate the total amount received from selling the tickets.
(iii) In 2006 the cost of organising the concert was $\$ 410$.

Calculate the percentage profit in 2006.
(c) In 2007, the number of tickets sold was again 210.

Adult tickets were \$ 2.60 each and student tickets were $\$ 1.40$ each.

The total amount received from selling the 210 tickets was $\$ 480$.

How many student tickets were sold?

## DO NOT DO ANY WORKING ON THIS QUESTION PAPER USE THE ANSWER BOOK OR PAPER PROVIDED

1 Beatrice has an income of $\$ 40000$ in one year.
(a) She pays:
no tax on the first $\$ 10000$ of her income;
$10 \%$ tax on the next $\$ 10000$ of her income;
$25 \%$ tax on the rest of her income.

Calculate
(i) the total amount of tax Beatrice pays,
(ii) the total amount of tax as a percentage of the $\$ 40000$
(b) Beatrice pays a yearly rent of $\$ 10800$.

After she has paid her tax, rent and bills, she has $\$ 12000$.
Calculate how much Beatrice spends on bills.
(c) Beatrice divides the $\$ 12000$ between shopping and saving in the ratio
shopping: saving $=5: 3$.
(i) Calculate how much Beatrice spends on shopping in one year.
(ii) What fraction of the original $\$ 40000$ does Beatrice save?

Give your answer in its lowest terms.
(d) The rent of $\$ 10800$ is an increase of $25 \%$ on her previous rent.

Calculate her previous rent.

For the diagram above write down
(a) the order of rotational symmetry,

> Answer(a)
(b) the number of lines of symmetry.
Answer(b)

2 Write down the next two prime numbers after 43.

3 Use your calculator to find the value of $\frac{\left(\cos 30^{\circ}\right)^{2}-\left(\sin 30^{\circ}\right)^{2}}{2\left(\sin 120^{\circ}\right)\left(\cos 120^{\circ}\right)}$.

> Answer

4 Simplify

$$
\frac{5}{8} x^{\frac{3}{2}} \div \frac{1}{2} x^{-\frac{5}{2}}
$$



Each of the lengths 24 cm and 18 cm is measured correct to the nearest centimetre.
Calculate the upper bound for the perimeter of the shape.

5 In 1970 the population of China was $8.2 \times 10^{8}$.
In 2007 the population of China was $1.322 \times 10^{9}$.
Calculate the population in 2007 as a percentage of the population in 1970.

14 Zainab borrows $\$ 198$ from a bank to pay for a new bed. The bank charges compound interest at $1.9 \%$ per month. Calculate how much interest she owes at the end of 3 months. Give your answer correct to 2 decimal places.

1 Chris goes to a shop to buy meat, vegetables and fruit.
(a) (i) The costs of the meat, vegetables and fruit are in the ratio meat $:$ vegetables $:$ fruit $=2: 2: 3$.

The cost of the meat is $\$ 2.40$.

Calculate the total cost of the meat, vegetables and fruit.

> Answer(a)(i) \$
[2]
(ii) Chris pays with a $\$ 20$ note.

What percentage of the $\$ 20$ has he spent?

(b) The masses of the meat, vegetables and fruit are in the ratio

$$
\text { meat }: \text { vegetables : fruit }=1: 8: 3 \text {. }
$$

The total mass is 9 kg .
Calculate the mass of the vegetables.
(c) Calculate the cost per kilogram of the fruit.

Answer(c) \$
[3]
(d) The cost of the meat, $\$ 2.40$, is an increase of $25 \%$ on the cost the previous week.

Calculate the cost of the meat the previous week.


8 Show that $\frac{7}{27}+1 \frac{7}{9}=2 \frac{1}{27}$.
Write down all the steps in your working.
Answer

9 When a car wheel turns once, the car travels 120 cm , correct to the nearest centimetre.
Calculate the lower and upper bounds for the distance travelled by the car when the wheel turns 20 times.


15 The air fare from Singapore to Stockholm can be paid for in Singapore dollars (S\$) or Malaysian Ringitts (RM).
One day the fare was $\mathrm{S} \$ 740$ or RM 1900 and the exchange rate was $\mathrm{S} \$ 1=\mathrm{RM} 2.448$.
How much less would it cost to pay in Singapore dollars?
Give your answer in Singapore dollars correct to the nearest Singapore dollar.

## Answer S\$

16 Simplify
(a) $\left(\frac{16}{81} x^{16}\right)^{\frac{1}{2}}$,
(b) $\frac{16 y^{10} \times 4 y^{-4}}{32 y^{7}}$.

17

> Answer(b)

For a small international school, the holiday destinations of the 255 students are shown in the table.
(a) Complete the table.
(b) What is the probability that a student chosen at random is a girl going on holiday to Europe?
Answer(b)

1 Write down the number which is 3.6 less than -4.7 .

> Answer

2 A plane took 1 hour and 10 minutes to fly from Riyadh to Jeddah.
The plane arrived in Jeddah at 2305.
At what time did the plane depart from Riyadh?
Answer

3 Calculate $\sqrt[3]{2.35^{2}-1.09^{2}}$.
Give your answer correct to 4 decimal places.

Answer

5 Show that $3 \frac{3}{4}+1 \frac{1}{3}=5 \frac{1}{12}$.
Write down all the steps in your working.
Answer

6 Write the following in order of size, smallest first.

$$
\begin{array}{llll}
\frac{20}{41} & \frac{80}{161} & 0.492 & 4.93 \%
\end{array}
$$

Answer $<$ ...............

7 In France, the cost of one kilogram of apricots is $€ 3.38$.
In the UK, the cost of one kilogram of apricots is $£ 4.39$.
$£ 1=€ 1.04$.
Calculate the difference between these prices.
Give your answer in pounds (£).

> Answer £

8 A large rectangular card measures 80 centimetres by 90 centimetres.
Maria uses all this card to make small rectangular cards measuring 40 millimetres by 15 millimetres.
Calculate the number of small cards.

1 (a) Hansi and Megan go on holiday.
The costs of their holidays are in the ratio Hansi : Megan $=7: 4$.
Hansi's holiday costs $\$ 756$.
Find the cost of Megan's holiday.

## Answer(a) \$

(b) In 2008, Hansi earned $\$ 7800$.
(i) He earned $15 \%$ more in 2009.

Calculate how much he earned in 2009.
Answer(b)(i) \$
(ii) In 2010, he earns $10 \%$ more than in 2009.

Calculate the percentage increase in his earnings from 2008 to 2010.
(c) Megan earned \$9720 in 2009. This was $20 \%$ more than she earned in 2008.

How much did she earn in 2008?

Answer(c) \$
(d) Hansi invested $\$ 500$ at a rate of $4 \%$ per year compound interest. Calculate the final amount he had after three years.
(b) A plane flies from London to Dubai and then to Colombo.

It leaves London at 0150 and the total journey takes 13 hours and 45 minutes.
The local time in Colombo is 7 hours ahead of London.
Find the arrival time in Colombo.

Answer(b)
[2]
(c) Another plane flies the 8710 km directly from London to Colombo at an average speed of $800 \mathrm{~km} / \mathrm{h}$.
How much longer did the plane in part (b) take to travel from London to Colombo? Give your answer in hours and minutes, correct to the nearest minute.


1 Thomas, Ursula and Vanessa share $\$ 200$ in the ratio

$$
\text { Thomas : Ursula : Vanessa }=3: 2: 5 \text {. }
$$

(a) Show that Thomas receives $\$ 60$ and Ursula receives $\$ 40$.

Answer(a)
(b) Thomas buys a book for $\$ 21$.

What percentage of his $\$ 60$ does Thomas have left?
(c) Ursula buys a computer game for $\$ 36.80$ in a sale.

The sale price is $20 \%$ less than the original price.
Calculate the original price of the computer game.


> Answer(c) \$
(d) Vanessa buys some books and some pencils.

Each book costs $\$ 12$ more than each pencil.
The total cost of 5 books and 2 pencils is $\$ 64.20$.
Find the cost of one pencil.

> Answer(d) \$

2 Make $h$ the subject of the formula $\quad g=\sqrt{h+i}$.

$$
\text { Answer } h=
$$

3 Find the value of $\left(\frac{9}{4}\right)^{-\frac{3}{2}}$, giving your answer as an exact fraction.

Answer

4 Showing all your working, calculate $\quad 1 \frac{1}{4} \div \frac{2}{3}-1 \frac{1}{3}$.


# EXTENDED MATHEMATICS 2002-2011 <br> CLASSIFIEDS SEQUENCIESnPATTERNS 

Compiled \& Edited Ву

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18 The first four terms of a sequence are
$\mathrm{T}_{1}=1^{2}$
$\mathrm{T}_{2}=1^{2}+2^{2}$
$\mathrm{T}_{3}=1^{2}+2^{2}+3^{2}$
$\mathrm{T}_{4}=1^{2}+2^{2}+3^{2}+4^{2}$.
(a) The $n$th term is given by $\mathrm{T}_{n}=\frac{1}{6} n(n+1)(2 n+1)$.

Work out the value of $\mathrm{T}_{23}$.

$$
\text { Answer }(a) \mathrm{T}_{23}=
$$

(b) A new sequence is formed as follows.

$$
\mathrm{U}_{1}=\mathrm{T}_{2}-\mathrm{T}_{1} \quad \mathrm{U}_{2}=\mathrm{T}_{3}-\mathrm{T}_{2} \quad \mathrm{U}_{3}=\mathrm{T}_{4}-\mathrm{T}_{3}
$$

(i) Find the values of $\mathrm{U}_{1}$ and $\mathrm{U}_{2}$.

$$
\begin{equation*}
\text { Answer(b)(i) } \mathrm{U}_{1}= \tag{2}
\end{equation*}
$$

$$
\text { and } U_{2}=
$$

(ii) Write down a formula for the $n$th term, $\mathrm{U}_{n}$.

$$
\text { Answer(b)(ii) } \mathrm{U}_{n}=
$$

(c) The first four terms of another sequence are
$\mathrm{V}_{1}=2^{2} \quad \mathrm{~V}_{2}=2^{2}+4^{2} \quad \mathrm{~V}_{3}=2^{2}+4^{2}+6^{2} \quad \mathrm{~V}_{4}=2^{2}+4^{2}+6^{2}+8^{2}$.
By comparing this sequence with the one in part (a), find a formula for the $n$th term, $\mathrm{V}_{n}$.

$$
\begin{equation*}
\text { Answer(c) } \mathrm{V}_{n}= \tag{2}
\end{equation*}
$$

$9 \quad$ A sequence is given by $\quad u_{1}=\sqrt{1}, \quad u_{2}=\sqrt{3}, \quad u_{3}=\sqrt{5}, \quad u_{4}=\sqrt{7}, \ldots$
(a) Find a formula for $\mathrm{u}_{n}$, the $n$th term.

$$
\operatorname{Answer}(a) \mathrm{u}_{n}=
$$

(b) Find $\mathrm{u}_{29}$.


12 (a) The $n$th term of a sequence is $n(n+1)$.
(i) Write the two missing terms in the spaces. 2, 6, ....... , 20, .........
(ii) Write down an expression in terms of $n$ for the $(n+1)$ th term.
Answer(a)(ii)
(iii) The difference between the $n$th term and the $(n+1)$ th term is $p n+q$.

Find the values of $p$ and $q$.

$$
\text { Answer(a)(iii) } p=
$$

$$
\begin{equation*}
q= \tag{2}
\end{equation*}
$$

(iv) Find the positions of the two consecutive terms which have a difference of 140 .
(b) A sequence $u_{1}, u_{2}, u_{3}, u_{4},$.

Answer(a)(iv) .......... and
[2] $u_{1}=2$,

For example, the third term is $u_{3}$ and $u_{3}=2 u_{1}+u_{2}=2 \times 2+3=7$.
So, the sequence is

## is $2,3,7, \mu_{4}, u_{5}, \ldots \ldots$

(i) Show that $u_{4}=13$.

Answer(b)(i)
(ii) Find the value of $u_{5}$.

$$
\operatorname{Answer}(b)\left(\text { ii) } u_{5}=\right.
$$

(iii) Two consecutive terms of the sequence are 3413 and 6827.

Find the term before and the term after these two given terms.

Answer(b)(iii) ................................... , 3413, 6827,

13


Pattern 1


Pattern 2


Pattern 3

The first three patterns in a sequence are shown above.
(a) Complete the table.

| Pattern number | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- |
| Number of dots | 5 |  |  |  |

(b) Find a formula for the number of dots, $d$, in the $n$th pattern.

$$
\text { Answer (b) } d=
$$

(c) Find the number of dots in the 60th pattern.

> Answer (c)

(d) Find the number of the pattern that has 89 dots.

14 A house was built in 1985 and cost $\$ 62000$.
It was sold in 2003 for $\$ 310000$.
(a) Work out the 1985 price as a percentage of the 2003 price.

Answer (a) $\qquad$ \% [2]
(b) Calculate the percentage increase in the price from 1985 to 2003.


Diagram 1


Diagram 2


Diagram 3

The first three diagrams in a sequence are shown above.
The diagrams are made up of dots and lines. Each line is one centimetre long.
(a) Make a sketch of the next diagram in the sequence.
(b) The table below shows some information about the diagrams.

| Diagram | 1 | 2 | 3 | 4 | $\cdots$ | $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Area | 1 | 4 | 9 | 16 | $\cdots$ | $x$ |
| Number of dots | 4 | 9 | 16 | $p$ | $\cdots$ |  |
| Number of one centimetre lines | 4 | 12 | 24 | $q$ | $\cdots \cdots$ | $y$ |

(i) Write down the values of $p$ and $q$.
(ii) Write down each of $x, y$ and $z$ in terms of $n$.
(c) The total number of one centimetre lines in the first $n$ diagrams is given by the expression

$$
\frac{2}{3} n^{3}+f n^{2}+g n
$$

(i) Use $n=1$ in this expression to show that $f+g=\frac{10}{3}$.
(ii) Use $n=2$ in this expression to show that $4 f+2 g=\frac{32}{3}$.
(iii) Find the values of $f$ and $g$.
(iv) Find the total number of one centimetre lines in the first 10 diagrams.

| 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 8 | 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 | 17 | 18 |
| 19 | 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 |

A 3 by 3 square

| $x$ | $b$ | $c$ |
| :--- | :--- | :--- |
| $d$ | $e$ | $f$ |
| $g$ | $h$ | $i$ | can be chosen from the 6 by 6 grid above.

(a) One of these squares is

| 8 | 9 | 10 |
| :---: | :---: | :---: |
| 14 | 15 | 16 |
| 20 | 21 | 22 |

In this square, $x=8, c=10, g=20$ and $i=22$.
For this square, calculate the value of
(i) $(i-x)-(g-c)$,
(ii) $c g-x i$.
(b)

| $x$ | $b$ | $c$ |
| :---: | :---: | :---: |
| $d$ | $e$ | $f$ |
| $g$ | $h$ | $i$ |

(i) $c=x+2$. Write down $g$ and $i$ in terms of $x$.
(ii) Use your answers to part(b)(i) to show that $(i-x)-(g-c)$ is constant.
(iii) Use your answers to $\operatorname{part}(\mathbf{b})(\mathbf{i})$ to show that $c g-x i$ is constant.
(c) The 6 by 6 grid is replaced by a 5 by 5 grid as shown.

| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 |

A 3 by 3 square | $\boldsymbol{x}$ | $\boldsymbol{b}$ | $\boldsymbol{c}$ |
| :--- | :--- | :--- |
| $\boldsymbol{d}$ | $\boldsymbol{e}$ | $\boldsymbol{f}$ |
| $\boldsymbol{g}$ | $\boldsymbol{h}$ | $\boldsymbol{i}$ | c can be chosen from the 5 by 5 grid.

For any 3 by 3 square chosen from this 5 by 5 grid, calculate the value of
(i) $(i-x)-(g-c)$,
(ii) $c g-x i$.
(d) A 3 by 3 square is chosen from an $n$ by $n$ grid.
(i) Write down the yalue of $(i-x)-(g-c)$.
(ii) Find $g$ and $i$ in terms of $x$ and $n$.
(iii) Find $c g-x i$ in its simplest form.
Diagram 1
Diagram 2
Diagram 3
Diagram 4

The first four terms in a sequence are $1,3,6$ and 10 .
They are shown by the number of dots in the four diagrams above.
(a) Write down the next four terms in the sequence.

> Answer(a)
(b) (i) The sum of the two consecutive terms 3 and 6 is 9 .

The sum of the two consecutive terms 6 and 10 is 16 .
Complete the following statements using different paiss of terms.
The sum of the two consecutive terms


The sum of the two consecutive terms $\square$ and ...........is is $\qquad$ .
(ii) What special name is given to these sums?

> Answer(b)(ii)
(c) (i) The formula for the $n$th term in the sequence $1,3,6,10 \ldots$ is $\frac{n(n+1)}{k}$, where $k$ is an integer.

Find the value of $k$.

$$
\begin{equation*}
\operatorname{Answer}(c)(\mathrm{i}) k= \tag{1}
\end{equation*}
$$

(ii) Test your formula when $n=4$, showing your working.

Answer (c)(ii)
(iii) Find the value of the 180 th term in the sequence.

Answer(c)(iii)
[1]
(d) (i) Show clearly that the sum of the $n$th and the $(n+1)$ th terms is $(n+1)^{2}$. Answer (d)(i)

(ii) Find the values of the two consecutive terms which have a sum of 3481 .


Diagram 1
1 white dot
5 black dots
6 lines


Diagram 2
4 white dots
7 black dots
14 lines


Diagram 3
9 white dots
9 black dots
26 lines


Diagram 4
16 white dots
11 black dots
42 lines

The four diagrams above are the first four of a pattern.
(a) Diagram 5 has been started below.

Complete this diagram and write down the information about the numbersof dots and lines.


Diagram 5
$\qquad$ white dots
$\qquad$ black dots
$\qquad$ lines
(b) Complete the information about the number of dots and lines in Diagram 8.

$$
\begin{array}{cl}
\text { Answer(b) } & . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~ w h i t e ~ d o t s ~ \\
\text {. } \\
& . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~ b l a c k ~ d o t s ~
\end{array}
$$

(c) Complete the information about the number of dots in Diagram $n$.

Give your answers in terms of $n$.

$$
\begin{aligned}
\text { Answer(c) } & . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~ w h i t e ~ d o t s ~ \\
& \\
& . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~ b l a c k ~ d o t s ~
\end{aligned}
$$

(d) The number of lines in diagram $n$ is $k\left(n^{2}+n+1\right)$.

Find
(i) the value of $k$,

(ii) the number of lines in Diagram 100 .

10


The diagrams show some polygons and their diagonals.
(a) Complete the table.

| Number of sides | Name of polygon | Total number of diagonals |
| :---: | :---: | :---: |
| 3 | triangle | 0 |
| 4 | quadrilateral | 2 |
| 5 | hexagon | 14 |
| 7 | heptagon |  |
| 8 |  |  |

(b) Write down the total number of diagonals in
(i) a decagon (a 10-sided polygon),
Answer(b)(i)
(ii) a 12-sided polygon.
Answer(b)(ii)
(c) A polygon with $n$ sides has a total of $\frac{1}{p} n(n-q)$ diagonals, where $p$ and $q$ are integers.
(i) Find the values of $p$ and $q$.

## Answer(c)(i) $p=$

$\qquad$

$$
q=
$$

(ii) Find the total number of diagonals in a polygon with 100 sides.

(iii) Find the number of sides of a polygon which has a total of 170 diagonals.

(d) A polygon with $n+1$ sides has 30 more diagonals than a polygon with $n$ sides. Find $n$.


Diagram 1 Diagram 2
Diagram 3
Diagram 4
The diagrams show squares and dots on a grid.
Some dots are on the sides of each square and other dots are inside each square.
The area of the square (shaded) in Diagram 1 is 1 unit $^{2}$.
(a) Complete Diagram 4 by marking all the dots.
(b) Complete the columns in the table below for Diagrams 4, 5 and $\boldsymbol{\eta}$.

| Diagram | 1 | 2 | 3 | 4 | 5 | ------ | $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of units of area | 1 | 4 | 9 |  |  | ------ |  |
| Number of dots inside the <br> square | 1 | 5 | 13 |  |  | ------ | $(n-1)^{2}+n^{2}$ |
| Number of dots on the sides <br> of the square | 4 | 8 | 12 |  |  | ------ |  |
| Total number of dots | 5 | 13 | 25 |  |  | ------ |  |

(c) For Diagram 200, find the number of dots
(i) inside the square,

> Answer(c)(i)
(ii) on the sides of the square.
Answer(c)(ii)
(d) Which diagram has 265 dots inside the square?

2 Write down the next term in each sequence.
(a) 1 ,
2,
4,
8 ,
16,
(b) 23,
19,
15,
11,
$\qquad$
11, 7,

10 (a) Write down the next two terms in each of the following sequences.
(i) 71,
64,
57,
50,
.......... .
(ii) -17 ,
-13,
-9,
-5,
........... ,
(b) The $n$th term of the sequence in part (a)(i) is $78-7 n$.

Find the value of the 15 th term.
Answer(b)
(c) Write down an expression for the $n$th term of the sequence in part (a)(ii).
(d) For one value of $n$, both sequences in part (a) have a term with the same value.

Use parts (b) and (c) to find
(i) the value of $n$,


$$
\operatorname{Answer}(d) \text { (i) } n=
$$

(ii) the value of this term.

9
-
Diagram 1
Diagram 2


Diagram 3
Diagram 4
Diagram 5

The Diagrams above form a pattern.
(a) Draw Diagram 5 in the space provided.
(b) The table shows the numbers of dots in some of the diagrams.

Complete the table.

| Diagram | 1 | 2 | 3 | 4 | 5 |  | 10 |  | $n$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of dots | 3 | 5 |  |  |  |  |  |  |  |

(c) What is the value of $n$ when the number of dots is 737 ?

> Answerg
(d) Complete the table which shows the total number of dots in consecutive pairs of diagrams.

For example, the total number of dots in Diagram 2 and Diagram 3 is 12 .

| Diagrams | 1 and 2 | 2 and 3 | 3 and 4 | 4 and 5 | 10 and 11 | $n$ and $n+1$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Total <br> number of <br> dots | 8 | 12 | 16 |  |  |  |  |

10 The first and the $n$th terms of sequences $A, B$ and $C$ are shown in the table below.
(a) Complete the table for each sequence.

|  | 1st term | 2nd term | 3rd term | 4th term | 5th term | $n$th term |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Sequence $A$ | 1 |  |  |  |  | $n^{3}$ |
| Sequence $B$ | 4 |  |  |  |  | $4 n$ |
| Sequence $C$ | 4 |  |  |  |  | $(n+1)^{2}$ |

(b) Find
(i) the 8th term of sequence $A$,

> Answer(b)(i)
(ii) the 12th term of sequence $C$.

## Answer(b)(ii)

(c) (i) Which term in sequence $A$ is equal to 15625 ?

> Answer (c)(i)
(ii) Which term in sequence $C$ is equal to 10000 ?
Answer(c)(ii)
(d) The first four terms of sequences $D$ and $E$ are shown in the table below.

Use the results from part (a) to find the 5 th and the $n$th terms of the sequences $D$ and $E$.

|  | 1st term | 2nd term | 3rd term | 4th term | 5th term | $n$th term |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Sequence $D$ | 5 | 16 | 39 | 80 |  |  |
| Sequence $E$ | 0 | 1 | 4 | 9 |  |  |

11 (a) (i) The first three positive integers 1,2 and 3 have a sum of 6 .
Write down the sum of the first 4 positive integers.
Answer(a)(i)
(ii) The formula for the sum of the first $n$ integers is $\frac{n(n+1)}{2}$.

Show the formula is correct when $n=3$.
Answer(a)(ii)
(iii) Find the sum of the first 120 positive integers.

(iv) Find the sum of the integers

(v) Find the sum of the even numbers
$2+4+6+$ $\qquad$ +800 .
(b) (i) Complete the following statements about the sums of cubes and the sums of integers.
$1^{3}=1$
$1=1$
$1^{3}+2^{3}=9$
$1+2=3$
$1^{3}+2^{3}+3^{3}=$
$1+2+3=$
............
$1^{3}+2^{3}+3^{3}+4^{3}=$ $\qquad$ $1+2+3+4=$ $\qquad$
(ii) The sum of the first 14 integers is 105 .

Find the sum of the first 14 cubes.
Answer(b)(ii)
(iii) Use the formula in $\mathbf{p a r t}(\mathbf{a})(i i)$ to write down a formula for the sum of the first $n$ cubes.
Answer(b)(iii)
(iv) Find the sum of the first 60 cubes.

(v) Find $n$ when the sum of the first $n$ cubes is 278784 .

3 The first five terms of a sequence are $4,9,16,25,36, \ldots$ Find
(a) the 10th term,
Answer (a)......................................... [1]
(b) the $n$th term.
Answer (b)......................................... [1]

4 Rearrange the quantities in order with the smallest first.

$$
\frac{1}{8} \%, \quad \frac{3}{2500}, \quad 0.00126
$$



0580/02/0581/02/O/N/03
1


The number of tennis balls $(T)$ in the diagram is given by the formula

$$
T=\frac{1}{2} n(n+1),
$$

where $n$ is the number of rows.
The diagram above has 4 rows.
How many tennis balls will there be in a diagram with 20 rows?

$$
1+2+3+4+5+\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots+n=\frac{n(n+1)}{2}
$$

(a) (i) Show that this formula is true for the sum of the first 8 natural numbers.
(ii) Find the sum of the first 400 natural numbers.
(b) (i) Show that $2+4+6+8+$ $\qquad$ $+2 n=n(n+1)$.
(ii) Find the sum of the first 200 even numbers.
(iii) Find the sum of the first 200 odd numbers.
(c) (i) Use the formula at the beginning of the question to find the sum of the first $2 n$ natural numbers.
(ii) Find a formula, in its simplest form, for


Show your working.

9 (a) The first five terms $P_{1}, P_{2}, P_{3}, P_{4}$ and $P_{5}$ of a sequence are given below.

| 1 | $=1=\mathrm{P}_{1}$ |
| :--- | :--- |
| $1+2$ | $=3=\mathrm{P}_{2}$ |
| $1+2+3$ | $=6=\mathrm{P}_{3}$ |
| $1+2+3+4$ | $=10=\mathrm{P}_{4}$ |
| $1+2+3+4+5$ | $=15=\mathrm{P}_{5}$ |

(i) Write down the next term, $\mathrm{P}_{6}$, in the sequence $1,3,6,10,15 \ldots$
Answer(a)(i)
(ii) The formula for the $n$th term of this sequence is

$$
\mathrm{P}_{n}=\frac{1}{2} n(n+1)
$$

Show this formula is true when $n=6$.

Answer (a)(ii)
(iii) Use the formula to find $\mathrm{P}_{50}$, the 50 th term of this sequence.
Answer(a)(iii)
(iv) Use your answer to part (iii) to find $3+6+9+12+15+$ $\qquad$ $+150$.

> Answer(a)(iv)
(v) Find $1+2+3+4+5+$. $\qquad$ $+150$.

$$
\begin{equation*}
\operatorname{Answer}(a)(\mathrm{v}) \tag{1}
\end{equation*}
$$

(vi) Use your answers to parts (iv) and (v) to find the sum of the numbers less than 150 which are not multiples of 3 .
Answer(a)(vi)
(b) The first five terms, $\mathrm{S}_{1}, \mathrm{~S}_{2}, \mathrm{~S}_{3}, \mathrm{~S}_{4}$ and $\mathrm{S}_{5}$ of a different sequence are given below.

$$
\begin{array}{ll}
(1 \times 1) & =1=\mathrm{S}_{1} \\
(1 \times 2)+(2 \times 1) & =4=\mathrm{S}_{2} \\
(1 \times 3)+(2 \times 2)+(3 \times 1) & =10=\mathrm{S}_{3} \\
(1 \times 4)+(2 \times 3)+(3 \times 2)+(4 \times 1) & =20=\mathrm{S}_{4} \\
(1 \times 5)+(2 \times 4)+(3 \times 3)+(4 \times 2)+(5 \times 1) & =35=\mathrm{S}_{5}
\end{array}
$$

(i) Work out the next term, $\mathrm{S}_{6}$, in the sequence $1,4,10,20,35 \ldots$
Answer(b)(i)
(ii) The formula for the $n$th term of this sequence is

$$
\mathrm{S}_{n}=\frac{1}{6} n(n+1)(n+2) .
$$

Show this formula is true for $n=6$.
Answer(b)(ii)
(iii) Find $(1 \times 20)+(2 \times 19)+(3 \times 18)$

(c) Show that $\mathrm{S}_{6}-\mathrm{S}_{5}=\mathrm{P}_{6}$, where $\mathrm{P}_{6}$ is your answer to part (a)(i).

Answer(c)
(d) Show by algebra that $\mathrm{S}_{n}-\mathrm{S}_{n-1}=\mathrm{P}_{n} . \quad\left[\mathrm{P}_{n}=\frac{1}{2} n(n+1)\right]$

Answer(d)

10 In all the following sequences, after the first two terms, the rule is to add the previous two terms to find the next term.
(a) Write down the next two terms in this sequence.
1
1
2
3
5
8
13
[1]
(b) Write down the first two terms of this sequence.
......... ......... 3 11 14
[2]
(c) (i) Find the value of $d$ and the value of $e$.
$2 d$
$e$
10

## Answer(c)(i) $d=$

$$
e=
$$

(ii) Find the value of $x$, the value of $y$ and the value of $z$.

$$
-33
$$



$$
\text { Answer(c)(ii) } \begin{aligned}
& =\text {......................................... } \\
y & =\ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~
\end{aligned} .
$$



Diagram 1


Diagram 2


Diagram 3


Diagram 4

The first four Diagrams in a sequence are shown above. Each Diagram is made from dots and one centimetre lines. The area of each small square is $1 \mathrm{~cm}^{2}$.
(a) Complete the table for Diagrams 5 and 6.

| Diagram | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Area $\left(\mathrm{cm}^{2}\right)$ | 2 | 6 | 12 | 20 |  |  |
| Number of dots | 6 | 12 | 20 | 30 |  |  |
| Number of one centimetre lines | 7 | 17 | 31 | 49 |  |  |

(b) The area of Diagram $n$ is $n(n+1) \mathrm{cm}^{2}$.
(i) Find the area of Diagram 50.

Answer(b)(i)
$\mathrm{cm}^{2}$
(ii) Which Diagram has an area of $930 \mathrm{~cm}^{2}$ ?
Answer(b)(ii)
(c) Find, in terms of $n$, the number of dots in Diagram $n$.
(d) The number of one centimetre lines in Diagram $n$ is $2 n^{2}+p n+1$.
(i) Show that $p=4$.

Answer(d)(i)
(ii) Find the number of one centimetre lines in Diagram 10.

(iii) Which Diagram has 337 one centimetre lines?

Answer(d)(iii)
(e) For each Diagram, the number of squares of area $1 \mathrm{~cm}^{2}$ is $A$, the number of dots is $D$ and the number of one centimetre lines is $L$.

Find a connection between $A, D$ and $L$ that is true for each Diagram.

## EXTENDED MATHEMATICS 2002-2011 <br> CLASSIFIEDS SETSnPROBABILITY



9 (a) Emile lost 2 blue buttons from his shirt.
A bag of spare buttons contains 6 white buttons and 2 blue buttons.
Emile takes 3 buttons out of the bag at random without replacement.
Calculate the probability that
(i) all 3 buttons are white,

(ii) exactly one of the 3 buttons is blue.


9 (a) Emile lost 2 blue buttons from his shirt.
A bag of spare buttons contains 6 white buttons and 2 blue buttons.
Emile takes 3 buttons out of the bag at random without replacement.
Calculate the probability that
(i) all 3 buttons are white,
(ii) exactly one of the 3 buttons is blue.

(b) There are 25 buttons in another bag.

This bag contains $x$ blue buttons.
Two buttons are taken at random without replacement.
The probability that they are both blue is $\frac{7}{100}$.


The diagram shows two sets of cards.
(a) One card is chosen at random from Set A and replaced.
(i) Write down the probability that the card chosen shows the letter M.
(ii) If this is carried out 100 times, write down the expected number of times the card chosen shows the letter M.
(b) Two cards are chosen at random, without replacement, from Set A.

Find the probability that both cards show the letter S.

Answer(b)
(c) One card is chosen at random from Set A and one card is chosen at random from Set B .

Find the probability that exactly one of the two cards shows the letter $U$.
Answer(c)
(d) A card is chosen at random, without replacement, from Set B until the letter shown is either I or U.


In the Venn diagram, $\mathscr{E}=\{$ students in a survey $\}, R=\{$ students who like rugby $\}$ and $F=\{$ students who like football $\}$.
$\mathrm{n}(\mathscr{E})=20$
$\mathrm{n}(R \cup F)=17$
$\mathrm{n}(R)=13$
$\mathrm{n}(F)=11$
(a) Find
(i) $\mathrm{n}(R \cap F)$,
Answer(a)(i)
(ii) $\mathrm{n}^{\prime}\left(\mathrm{R}^{\prime} \cap F\right)$.
Answer(a)(ii)
(b) A student who likes rugby is chosen at random.

Find the probability that this student also likes football.

16 In a survey of 60 cars, the type of fuel that they use is recorded in the table below.
Each car only uses one type of fuel.

| Petrol | Diesel | Liquid Hydrogen | Electricity |
| :---: | :---: | :---: | :---: |
| 40 | 12 | 2 | 6 |

(a) Write down the mode.

$$
\text { Answer }(a)
$$

(b) Olav drew a pie chart to illustrate these figures.

Calculate the angle of the sector for Diesel.

(c) Calculate the probability that a car chosen at random uses Electricity.

Write your answer as a fraction in its simplest form.


Diagram 1


Diagram 2
(a) In Diagram 1, shade the area which represents $A \cup B^{\prime}$.
(b) Describe in set notation the shaded area in Diagram 2.
Answer (b)

10 In a flu epidemic $45 \%$ of people have a sore throat.
If a person has a sore throat the probability of not having flu is 0.4
If a person does not have a sore throat the probability of having flu is, 0.2 .


Calculate the probability that a person chosen at random has flu.

3 Paula and Tarek take part in a quiz.
The probability that Paula thinks she knows the answer to any question is 0.6.
If Paula thinks she knows, the probability that she is correct is 0.9 .
Otherwise she guesses and the probability that she is correct is 0.2 .
(a) Copy and complete the tree diagram.

(b) Find the probability that Paula
(i) thinks she knows the answer and is correct,
(ii) gets the correct answer.
(c) The probability that Tarek thinks he knows the answer to any questionis 0.55 . If Tarek thinks he knows, he is always correct.
Otherwise he guesses and the probability that he is correct is $0.2 \overbrace{}^{\circ}$
(i) Draw a tree diagram for Tarek. Write all the probabilifies on your diagram.
(ii) Find the probability that Tarek gets the correct answer.
(d) There are 100 questions in the quiz.

Estimate the number of correct answers given by
(i) Paula,
(ii) Tarek.

3 There are 2 sets of road signals on the direct 12 kilometre route from Acity to Beetown.
The signals say either "GO" or "STOP".
The probabilities that the signals are "GO" when a car arrives are shown in the tree diagram.
(a) Copy and complete the tree diagram for a car driver travelling along this route.

(c) With no stops, Damon completes the 12 kilometre journey at an average speed of 40 kilometres per hour.
(i) Find the time taken in minutes for this journey.
(ii) When Damon has to stop at a signal it adds 3 minutes to this journey time.

Calculate his average speed, in kilometres per hour, if he stops at both road signals.
(d) Elsa takes a different route from Acity to Beetown.

This route is 15 kilometres and there are no road signals.
Elsa's average speed for this journey is 40 kilometres per hour.
Find
(i) the time taken in minutes for this journey,
(ii) the probability that Damon takes more time than this on his 12 kilometre journey.

7 (a) There are 30 students in a class.
20 study Physics, 15 study Chemistry and 3 study neither Physics nor Chemistry.

(i) Copy and complete the Venn diagram to show this information.
(ii) Find the number of students who study both Physics and Chemistry.
(iii) A student is chosen at random. Find the probability that the student studies Physics but not Chemistry.
(iv) A student who studies Physics is chosen at random. Find the probability that this student does not study Chemistry.
(b)

Bag A contains 6 white beads and 3 black beads.
Bag B contains 6 white beads and 4 black beads.
One bead is chosen at random from each bag.
Find the probability that
(i) both beads are black,
(ii) at least one of the two beads is white.

The beads are not replaced.
A second bead is chosen at random from each bag.
Find the probability that
(iii) all four beads are white,
(iv) the beads are not all the same colour.

18 Revina has to pass a written test and a driving test before she can drive a car on her own. The probability that she passes the written test is 0.6 .
The probability that she passes the driving test is 0.7 .
(a) Complete the tree diagram below.

$$
\text { Written test } \quad \text { Driving test }
$$


(b) Calculate the probability that Revina passes only one of the two tests.

8 On the Venn diagrams shade the regions
(a) $A^{\prime} \cap C^{\prime}$,

(b) $(A \cup C) \cap B$.


3 (a)


Bag A


Bag B

Nadia must choose a ball from Bag A or from Bag B.
The probability that she chooses Bag A is $\frac{2}{3}$.
Bag A contains 5 white and 3 black balls.
Bag B contains 6 white and 2 black balls.
The tree diagram below shows some of this information.

(i) Find the values of $p, q, r$ and $s$.
(ii) Find the probability that Nadia chooses Bag A and then a white ball.
(iii) Find the probability that Nadia chooses a white ball.
(b) Another bag contains 7 green balls and 3 yellow balls.

Sani takes three balls out of the bag, without replacement.
(i) Find the probability that all three balls he chooses are yellow.
(ii) Find the probability that at least one of the three balls he chooses is green.

| First | Second | Third |
| :--- | :--- | :--- |
| Calculator | Calculator | Calculator |



$$
\begin{aligned}
\mathrm{F} & =\text { faulty } \\
\mathrm{NF} & =\text { not faulty }
\end{aligned}
$$

The tree diagram shows a testing procedure on calculators, taken from alarge batch.
Each time a calculator is chosen at random, the probability that it is faulty (F) is $\frac{1}{20}$.
(a) Write down the values of $p$ and $q$.

Answer(b)(i)
(ii) exactly one is faulty.
(c) If exactly one out of two calculators tested is faulty, then a third calculator is chosen at random.

Calculate the probability that exactly one of the first two calculators is faulty and the third one is faulty.
(d) The whole batch of calculators is rejected
either if the first two chosen are both faulty
or if a third one needs to be chosen and it is faulty.
Calculate the probability that the whole batch is rejected.

> Answer(d)
(e) In one month, 1000 batches of calculators are tested in this way.

How many batches are expected to be rejected?

A

B

Box A contains 3 black balls and 1 white ball.
Box B contains 3 black balls and 2 white balls.
(a) A ball can be chosen at random from either box.

Complete the following statement.
There is a greater probability of choosing a white ball from Box

Explain your answer.

Answer (a)
(b) Abdul chooses a box and then chooses a ball from this box at random.

The probability that he chooses box A is $\frac{2}{3}$.
(i) Complete the tree diagram by writing the four probabilities in the empty spaces.

(ii) Find the probability that Abdul chooses box A and a black ball.
Answer(b)(ii)
(iii) Find the probability that Abdul chooses a black ball.
Answer(b)(iii)
(c) Tatiana chooses a box and then chooses two balls from this box at random (without replacement).

The probability that she chooses box A is $\frac{2}{3}$.
Find the probability that Tatiana chooses two white balls.



The diagram shows a spinner with six numbered sections.
Some of the sections are shaded.
Each time the spinner is spun it stops on one of the six sections.
It is equally likely that it stops on any one of the sections.
(a) The spinner is spun once.

Find the probability that it stops on
(i) a shaded section,
(ii) a section numbered 1 ,
Answer(a)(iii)
(iv) a shaded section or a section numbered 1 .
Answer(a)(iv)
(b) The spinner is now spun twice.

Find the probability that the total of the two numbers is
(i) 20 ,
Answer(b)(i)
(ii) 11 .

(c) (i) The spinner stops on a shaded section.

Find the probability that this section is numbered 2 .
(ii) The spinner stops on a section numbered 2 .

Find the probability that this section is shaded.

> Answer(c)(ii)
(d) The spinner is now spun until it stops on a section numbered 2 .

The probability that this happens on the $n$th spin is $\frac{16}{243}$.
Find the value of $n$.

$$
\operatorname{Answer}(d) n=
$$



The diagram shows a circular board, divided into 10 numbered sectors.

When the arrow is spun it is equally likely to stop in any sector
(a) Complete the table below which shows the probability of the arrow stopping at each number.

(b) The arrow is spun once.

Find
(i) the most likely number,
Answer(b)(i)
(ii) the probability of a number less than 4.
Answer(b)(ii)
(c) The arrow is spun twice.

Find the probability that
(i) both numbers are 2 ,

> Answer(c)(i)
(ii) the first number is 3 and the second number is 4 ,

Answer(c)(ii)
(iii) the two numbers add up to 4 .
(d) The arrow is spun several times until it stops at a number 4.

Find the probability that this happens on the third spin.

2 Shade the required region on each Venn diagram.


15 A teacher asks 36 students which musical instruments they play.
$P=\{$ students who play the piano $\}$
$G=\{$ students who play the guitar $\}$
$D=\{$ students who play the drums $\}$

The Venn diagram shows the results.

(a) Find the value of $x$.
(b) A student is chosen at random.

Find the probability that this student
(i) plays the drums but not the guitar,

Answer(b)(i)

Answer(b)(ii) $\qquad$
(c) A student is chosen at random from those who play the guitar.

Find the probability that this student plays no other instrument.

## 2 In this question give all your answers as fractions.

The probability that it rains on Monday is $\frac{3}{5}$.
If it rains on Monday, the probability that it rains on Tuesday is $\frac{4}{7}$.
If it does not rain on Monday, the probability that it rains on Tuesday is $\frac{5}{7}$.
(a) Complete the tree diagram.

Monday
Tuesday

(b) Find the probability that it rains
(i) on both days,
(ii) on Monday but not on Tuesday,
Answer (b)(i)

Answer(b)(ii)
(iii) on only one of the two days.
Answer(b)(iii)
(c) If it does not rain on Monday and it does not rain on Tuesday, the probability that it does not rain on Wednesday is $\frac{1}{4}$.
Calculate the probability that it rains on at least one of the three days.

7 Katrina puts some plants in her garden.
The probability that a plant will produce a flower is $\frac{7}{10}$.
If there is a flower, it can only be red, yellow or orange.
When there is a flower, the probability it is red is $\frac{2}{3}$ and the probability it is yellow is $\frac{1}{4}$.
(a) Draw a tree diagram to show all this information.

Label the diagram and write the probabilities on each branch.
Answer(a)
(b) A plant is chosen at random.

Find the probability that it will not produce a yellow flower.

> Answer(b)

(c) If Katrina puts 120 plants in her garden, how many orange flowers would she expect?


A wheel is divided into 10 sectors numbered 1 to 10 as shown in the diagram.
The sectors 1, 2, 3 and 4 are shaded.
The wheel is spun and when it stops the fixed arrow points to one of the sectors. (Each sector is equally likely.)
(a) The wheel is spun once so that one sector is selected. Find the probability that
(i) the number in the sector is even,
(ii) the sector is shaded,
(iii) the number is eyen or the sector is shaded,
(iv) the number is odd and the sector is shaded.
(b) The wheel is spun twice so that each time a sector is selected. Find the probability that
(i) both sectors are shaded,
(ii) one sector is shaded and one is not,
(iii) the sum of the numbers in the two sectors is greater than 20,
(iv) the sum of the numbers in the two sectors is less than 4,
(v) the product of the numbers in the two sectors is a square number.

20 A gardener plants seeds from a packet of 25 seeds.
14 of the seeds will give red flowers and 11 will give yellow flowers.
The gardener chooses two seeds at random.
(a) Write the missing probabilities on the tree diagram below.

First seed
Second seed

(b) What is the probability that the gardener chooses two seeds which will give
(i) two red flowers,
(ii) two flowers of a different colour?

4 (a) All 24 students in a class are asked whether they like football and whether they like basketball. Some of the results are shown in the Venn diagram below.

$\mathscr{E}=\{$ students in the class $\}$.
$F=\{$ students who like football $\}$.
$B=\{$ students who like basketball $\}$.
(i) How many students like both sports?
(ii) How many students do not like either sport?
(iii) Write down the value of $\mathrm{n}(F \cup B)$.
(iv) Write down the value of $\mathrm{n}\left(F^{\prime} \cap B\right)$.
(v) A student from the class is selected at random.

What is the probability that this student likes basketball?
(vi) A student who likes football is selected at random.

What is the probability that this student likes basketball?
(b) Two students are selected at random from a group of 10 boy§ and 12 girls.

Find the probability that
(i) they are both girls,
(ii) one is a boy and one is a girl.
(i) Write down, as fractions, the values of $s, t$ and $u$.
(ii) Calculate the probability that it rains on both days.
(iii) Calculate the probability that it will not rain tomorrow.
(b) Each time Christina throws a ball at a target, the probability that she hits the target is $\frac{1}{3}$.

She throws the ball three times.
Find the probability that she hits the target
(i) three times,
(ii) at least once.
(c) Each time Eduardo throws a ball at the target, the probability that he hits the target is $\frac{1}{4}$.

He throws the ball until he hits the target.
Find the probability that he first hits the target with his
(i) 4th throw,
(ii) $n$th throw.

9 In a survey, 100 students are asked if they like basketball $(B)$, football $(F)$ and swimming $(S)$.
The Venn diagram shows the results.


42 students like swimming.
40 students like exactly one sport.
(a) Find the values of $p, q$ and $r$.
(b) How many students like
(i) all three sports,
(ii) basketball and swimming but not football?
(c) Find
(i) $\mathrm{n}\left(B^{\prime}\right)$,
(ii) $\mathrm{n}\left((B \cup F) \cap S^{\prime}\right)$.
(d) One student is chosen at random from the 100 students.

Find the probability that the student
(i) only likes swimming,
(ii) likes basketball but not swimming.
(e) Two students are chosen at random from those who like basketball.

Find the probability that they each like exactly one other sport.


Six cards are numbered $1,1,6,7,11$ and 12.
In this question, give all probabilities as fractions.
(a) One of the six cards is chosen at random.
(i) Which number has a probability of being chosen of $\frac{1}{3}$ ?
Answer(a)(i)
(ii) What is the probability of choosing a card with a number which is smaller than at least three of the other numbers?
(b) Two of the six cards are chosen at random, without replacement.,

Find the probability that
(i) they are both numbered 1 ,

> Answer(b)(i)

(ii) the total of the two numbers is 18 ,
(iii) the first number is not a 1 and the second number is a 1 .
Answer(b)(iii)
(c) Cards are chosen, without replacement, until a card numbered 1 is chosen.

Find the probability that this happens before the third card is chosen.

(d) A seventh card is added to the six cards shown in the diagram. The mean value of the seven numbers on the cards is 6 .

Find the number on the seventh card
Answer(d)

9 A bag contains 7 red sweets and 4 green sweets.
Aimee takes out a sweet at random and eats it. She then takes out a second sweet at random and eats it.
(a) Complete the tree diagram.

$$
\text { First sweet } \quad \text { Second sweet }
$$


(b) Calculate the probability that Aimee has taken
(i) two red sweets,

(ii) one sweet of each colour.
(c) Aimee takes a third sweet at random.

Calculate the probability that she has taken
(i) three red sweets,
(ii) at least one red sweet.


# EXTENDED MATHEMATICS 2002-2011 CLASSIFIEDS STATISTICS 

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12


The scatter diagram shows the marks obtained in a Mathematics test and the marks obtained in an English test by 15 students.
(a) Describe the correlation.
(b) The mean for the Mathematics test is 47.3 .

The mean for the English test is 30.3 .
Plot the mean point $(47.3,30.3)$ on the scatter diagram above.
(c) (i) Draw the line of best fit on the diagram above.
(ii) One student missed the English test.

She received 45 marks in the Mathematics test.
Use your line to estimate the mark she might have gained in the English test.
Answer(c)(ii)

16 In a survey of 60 cars, the type of fuel that they use is recorded in the table below.
Each car only uses one type of fuel.

| Petrol | Diesel | Liquid Hydrogen | Electricity |
| :---: | :---: | :---: | :---: |
| 40 | 12 | 2 | 6 |

(a) Write down the mode.

> Answer(a)
(b) Olav drew a pie chart to illustrate these figures.

Calculate the angle of the sector for Diesel.


Answer(b)
(c) Calculate the probability that a car chosen at random uses Electricity.

Write your answer as a fraction in its simplest form.

3 The table shows information about the heights of 120 girls in a swimming club.

| Height $(h$ metres $)$ | Frequency |
| :---: | :---: |
| $1.3<h \leqslant 1.4$ | 4 |
| $1.4<h \leqslant 1.5$ | 13 |
| $1.5<h \leqslant 1.6$ | 33 |
| $1.6<h \leqslant 1.7$ | 45 |
| $1.7<h \leqslant 1.8$ | 19 |
| $1.8<h \leqslant 1.9$ | 6 |

(a) (i) Write down the modal class.
(ii) Calculate an estimate of the mean height. Show all of your working.

(b) Girls from this swimming club are chosen at random to swim in a race.

Calculate the probability that
(i) the height of the first girl chosen is more than 1.8 metres,

> Answer(b)(i)
(ii) the heights of both the first and second girl chosen are 1.8 metres or less.
(c) (i) Complete the cumulative frequency table for the heights.

| Height ( $h$ metres $)$ | Cumulative frequency |
| :---: | :---: |
| $h \leqslant 1.3$ | 0 |
| $h \leqslant 1.4$ | 4 |
| $h \leqslant 1.5$ | 17 |
| $h \leqslant 1.6$ | 50 |
| $h \leqslant 1.7$ |  |
| $h \leqslant 1.8$ | 114 |
| $h \leqslant 1.9$ |  |

(ii) Draw the cumulative frequency graph on the grid.

(d) Use your graph to find
(i) the median height,

> Answer(d)(i) ..................................... m [1]
(ii) the 30th percentile.

Answer(d)(ii)
m [1]

5 (a) The times, $t$ seconds, for 200 people to solve a problem are shown in the table.

| Time $(t$ seconds $)$ | Frequency |
| :---: | :---: |
| $0<t \leqslant 20$ | 6 |
| $20<t \leqslant 40$ | 12 |
| $40<t \leqslant 50$ | 20 |
| $50<t \leqslant 60$ | 37 |
| $60<t \leqslant 70$ | 42 |
| $70<t \leqslant 80$ | 50 |
| $80<t \leqslant 90$ | 28 |
| $90<t \leqslant 100$ | 5 |

Calculate an estimate of the mean time.

## Answer(a)

$\qquad$
(b) (i) Complete the cumulative frequency table for this data.

| Time <br> $(t$ seconds $)$ | $t \leqslant 20$ | $t \leqslant 40$ | $t \leqslant 50$ | $t \leqslant 60$ | $t \leqslant 79$ | $t \leqslant 80$ | $t \leqslant 90$ | $t \leqslant 100$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cumulative <br> Frequency | 6 | 18 | 38 |  | 0 | 167 |  |  |

(ii) Draw the cumulative frequency graph on the grid opposite to show this data.
(c) Use your cumulative frequency graph to find
(i) the median time,

> Answer(c)(i)
(ii) the lower quartile,
Answer(c)(ii)
(iii) the inter-quartile range,
Answer(c)(iii)
$\qquad$
(iv) how many people took between 65 and 75 seconds to solve the problem,
Answer(c)(iv)
(v) how many people took longer than 45 seconds to solve the problem.
Answer(c)(v)


7 The times, $t$ minutes, taken for 200 students to cycle one kilometre are shown in the table.

| Time ( $t$ minutes) | $0<t \leqslant 2$ | $2<t \leqslant 3$ | $3<t \leqslant 4$ | $4<t \leqslant 8$ |
| :---: | :---: | :---: | :---: | :---: |
| Frequency | 24 | 68 | 72 | 36 |

(a) Write down the class interval that contains the median.
(b) Calculate an estimate of the mean.

Show all your working.


Answer(b)
$\min [4]$
(c) (i) Use the information in the table opposite to complete the cumulative frequency table.

| Time ( $t$ minutes $)$ | $t \leqslant 2$ | $t \leqslant 3$ | $t \leqslant 4$ | $t \leqslant 8$ |
| :---: | :---: | :---: | :---: | :---: |
| Cumulative frequency | 24 |  |  | 200 |

(ii) On the grid, draw a cumulative frequency diagram.

(iii) Use your diagram to find the median, the lower quartile and the inter-quartile range.

```
Answer(c)(iii) Median =
    ......................... min
    Lower quartile =
```

$\qquad$

``` min
Inter-quartile range =
min [3]
```

9 (a) The number of people living in six houses is

$$
3,8,4, x, y \text { and } z .
$$

The median is $7 \frac{1}{2}$.
The mode is 8 .
The mean is 7 .
Find a value for each of $x, y$ and z .
(b) The grouped frequency table below shows the amount $(\$ A)$ spent on travel by a number of students.

| Cost of travel $(\$ A)$ | $0<A \leqslant 10$ | $10<A \leqslant 20$ | $20<A \leqslant 40$ |
| :---: | :---: | :---: | :---: |
| Frequency | 15 | $m$ | $n$ |

(i) Write down an estimate for the total amount in terms of $m$ and $n$.
(ii) The calculated estimate of the mean amount is $\$ 13$ exactly.

Write down an equation containing $m$ and $n$.
Show that it simplifies to $2 m+17 n=120$.
(iii) A student drew a histogram to represent this data.

The area of the rectangle representing the $0<A \leqslant 10$ group was equal to the sum of the areas of the other two rectangles.

Explain why $m+n=15$.
(iv) Find the values of $m$ and $n$ by solving the simultaneous equations

$$
2 m+17 n=120,
$$

$$
\begin{equation*}
m+n=15 \text {. } \tag{3}
\end{equation*}
$$

13 A doctor's patients are grouped by age, as shown in the table and the histogram below.

| Age $(x$ years $)$ | $0 \leqslant x<10$ | $10 \leqslant x<30$ | $30 \leqslant x<60$ | $60 \leqslant x<100$ |
| :--- | :---: | :---: | :---: | :---: |
| Number of patients | 300 | 600 |  | 880 |


(a) Complete the following:
$1 \mathrm{~cm}^{2}$ represents

(b) Use the histogram to fill in the blank in the table.
(c) Draw the missing two rectangles to complete the histogram.

14 (a) Multiply $\left(\begin{array}{rr}5 & 4 \\ -3 & -2\end{array}\right)\left(\begin{array}{rrr}2 & 1 & -4 \\ 0 & 3 & 6\end{array}\right)$.

(b) Find the inverse of $\left(\begin{array}{rr}5 & 4 \\ -3 & -2\end{array}\right)$.

$$
\text { Answer (b) } \quad(
$$

## 8 Answer the whole of this question on a sheet of graph paper.

In a survey, 200 shoppers were asked how much they had just spent in a supermarket.
The results are shown in the table.

| Amount $(\$ x)$ | $0<x \leqslant 20$ | $20<x \leqslant 40$ | $40<x \leqslant 60$ | $60<x \leqslant 80$ | $80<x \leqslant 100$ | $100<x \leqslant 140$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of shoppers | 10 | 32 | 48 | 54 | 36 | 20 |

(a) (i) Write down the modal class.
(ii) Calculate an estimate of the mean amount, giving your answer correct to 2 decimal places.
(b) (i) Make a cumulative frequency table for these 200 shoppers.
(ii) Using a scale of 2 cm to represent $\$ 20$ on the horizontal axis and 2 cm to represent 20 shoppers on the vertical axis, draw a cumulative frequency diagram for this data. [4]
(c) Use your cumulative frequency diagram to find
(i) the median amount,
(ii) the upper quartile,
(iii) the interquartile range,
(iv) how many shoppers spent at least $\$ 75$.

3 The depth, $d$ centimetres, of a river was recorded each day during a period of one year (365 days). The results are shown by the cumulative frequency curve.
cumulative frequency

(a) Use the cumulative frequency curve to find
(i) the median depth
(ii) the inter-quartile range,
depth, $d$ (cm)
(iii) the depth at the $40^{\text {th }}$ percentile,
(iv) the number of days when the depth of the river was at least 25 cm .
(b)

| $d$ | $0<d \leqslant 10$ | $10<d \leqslant 20$ | $20<d \leqslant 30$ | $30<d \leqslant 40$ | $40<d \leqslant 50$ | $50<d \leqslant 60$ | $60<d \leqslant 70$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of days | 17 | 41 | 62 | 98 | 85 | $p$ | $q$ |

(i) Show that $p=47$ and $q=15$.
(ii) Use the information in the table and the values of $p$ and $q$ to calculate an estimate of the mean depth of the river.
$\square$
(c) The following information comes from the table in part (b).

| $d$ | $0<d \leqslant 20$ | $20<d \leqslant 40$ | $40<d \leqslant 70$ |
| :---: | :---: | :---: | :---: |
| Number of days | 58 | 160 | 147 |

A histogram was drawn to show this information.
The height of the column for the interval $20<d \leqslant 40$ was 8 cm .
Calculate the height of each of the other two columns.
[Do not draw the histogram.]

7 The speeds ( $v$ kilometres/hour) of 150 cars passing a $50 \mathrm{~km} / \mathrm{h}$ speed limit sign are recorded. A cumulative frequency curve to show the results is drawn below.

(a) Use the graph to find
(i) the median speed,
(ii) the inter-quartile range of the speeds,
(iii) the number of cars travelling with speeds of more than $50 \mathrm{~km} / \mathrm{h}$.
(b) A frequency table showing the speeds of the cars is

| Speed $(v \mathrm{~km} / \mathrm{h})$ | $30<v \leqslant 35$ | $35<v \leqslant 40$ | $40<v \leqslant 45$ | $45<v \leqslant 50$ | $50<v \leqslant 55$ | $55<v \leqslant 60$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 10 | 17 | 33 | 42 | $n$ | 16 |

(i) Find the value of $n$.
(ii) Calculate an estimate of the mean speed.
(c) Answer this part of this question on a sheet of graph paper.

Another frequency table for the same speeds is

| Speed $(v \mathrm{~km} / \mathrm{h})$ | $30<v \leqslant 40$ | $40<v \leqslant 55$ | $55<v \leqslant 60$ |
| :---: | :---: | :---: | :---: |
| Frequency | 27 | 107 | 16 |

Draw an accurate histogram to show this information.
Use 2 cm to represent 5 units on the speed axis and 1 cm to represent 1 uniton the frequency density axis (so that $1 \mathrm{~cm}^{2}$ represents 2.5 cars).

8

$$
\mathrm{f}(x)=x^{2}-4 x+3
$$

and $\quad \mathrm{g}(x)=2 x-1$.
(a) Solve $\mathrm{f}(x)=0$.
(b) Find $\mathrm{g}^{-1}(x)$.
(c) Solve $\mathrm{f}(x)=\mathrm{g}(x)$, giving your answers correct to 2 decimal places.
(d) Find the value of $g f(-2)$.
(e) Find $\mathrm{fg}(x)$. Simplify your answer.

Kristina asked 200 people how much water they drink in one day.
The table shows her results.

| Amount of water ( $x$ litres $)$ | Number of people |
| :---: | :---: |
| $0<x \leqslant 0.5$ | 8 |
| $0.5<x \leqslant 1$ | 27 |
| $1<x \leqslant 1.5$ | 45 |
| $1.5<x \leqslant 2$ | 50 |
| $2<x \leqslant 2.5$ | 39 |
| $2.5<x \leqslant 3$ | 21 |
| $3<x \leqslant 3.5$ | 7 |
| $3.5<x \leqslant 4$ | 3 |

(a) Write down the modal interval.
(d) Using a scale of 4 cm to 1 litre of water on the horizontal axis and 1 cm to 10 people on the vertical axis, draw the cumulative frequency graph.
(e) Use your cumulative frequency graph to find
(i) the median,
(ii) the $40^{\text {th }}$ percentile,
(iii) the number of people who drink at least 2.6 litres of water.
(f) A doctor recommends that a person drinks at least 1.8 litres of water each day. What percentage of these 200 people do not drink enough water?

4


200 people record the number of hours they work in a week.
The cumulative frequency graph shows this information.
(a) Use the graph to find
(i) the median,
(ii) the upper quartile,
(iii) the inter-quartile range,
(iv) the number of people who work more than 60 hours in a week.
(b) Omar uses the graph to make the following frequency table.

| Hours <br> worked ( $h$ ) | $0<h \leqslant 10$ | $10<h \leqslant 20$ | $20<h \leqslant 30$ | $30<h \leqslant 40$ | $40<h \leqslant 50$ | $50<h \leqslant 60$ | $60<h \leqslant 70$ | $70<h \leqslant 80$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 12 | 34 | 36 | 30 | 38 | 30 | $p$ | $q$ |

(i) Use the graph to find the yalues of $p$ and $q$.
(ii) Calculate an estimate of the mean number of hours worked in a week.
(c) Shalini uses the graph to make a different frequency table.

| Hours worked (h) | $0<h \leqslant 30$ | $30<h \leqslant 40$ | $40<h \leqslant 50$ | $50<h \leqslant 80$ |
| :---: | :---: | :---: | :---: | :---: |
| Frequency | 82 | 30 | 38 | 50 |

When she draws a histogram, the height of the column for the interval $30<h \leqslant 40$ is 9 cm .

Calculate the height of each of the other three columns.

2 A normal die, numbered 1 to 6 , is rolled 50 times.

The results are shown in the frequency table.


| Score | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 15 | 10 | 7 | 5 | 6 | 7 |

(a) Write down the modal score.
Answer(a)
(b) Find the median score.
(c) Calculate the mean score.

Answer(c)
(d) The die is then rolled another 10 times.

The mean score for the 60 rolls is 2.95 .
Calculate the mean score for the extra 10 rolls.

9 The heights of 100 students are measured.
The results have been used to draw this cumulative frequency diagram.

Cumulative
frequency

(a) Find
(i) the median height,
Answer(a)(i)
$\qquad$
(ii) the lower quartile,
Answer(a)(ii)
$\qquad$
(iii) the inter-quartile range,
Answer(a)(iii)
$\qquad$
(iv) the number of students with a height greater than 177 cm .

(b) The frequency table shows the information about the 100 students who were measured.

(i) Use the cumulative frequency diagram to complete the table above.
(ii) Calculate an estimate of the mean height of the 100 students.

240 students are asked about the number of people in their families.
The table shows the results.

| Number of people in family | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 1 | 1 | 17 | 12 | 6 | 3 |

(a) Find
(i) the mode,

> Answer(a)(i)
(ii) the median,

Answer(a)(ii)
(iii) the mean.

Answer(a)(iii)
(b) Another $n$ students are asked about the number of people in their families.

The mean for these $n$ students is 3 .
Find, in terms of $n$, an expression for the mean number for all $(40+n)$ students.
(i) Use the information from the histogram to complete the frequency table.

| Number of <br> hours $(h)$ | $0<h \leqslant 5$ | $5<h \leqslant 8$ | $8<h \leqslant 10$ | $10<h \leqslant 12$ | $12<h \leqslant 16$ | $16<h \leqslant 20$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency |  |  |  | 20 | 24 | 10 |

(ii) Use the information in this table to calculate an estimate of the mean number of hours. Show your working.

$\qquad$ hours

830 students took a vocabulary test.
The marks they scored are shown below.

| 7 | 8 | 5 | 8 | 3 | 2 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 6 | 6 | 3 | 3 | 6 | 2 |
| 7 | 1 | 5 | 10 | 2 | 6 |
| 6 | 5 | 8 | 1 | 2 | 7 |
| 3 | 1 | 5 | 3 | 10 | 3 |

(a) Complete the frequency table below.

The first five frequencies have been completed for you.
You may use the tally column to help you.

| Mark | Tally | Frequency |
| :---: | :---: | :---: |
| 1 |  | $\square$ |
| 2 |  |  |
| 3 | $\square$ | P 6 |
| 4 |  | 0 |
| 5 | $\checkmark$ | 4 |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |

(b) (i) Find the range.

> Answer(b)(i)
(ii) Write down the mode.

Answer(b)(ii)
(iii) Find the median.

> Answer(b)(iii)
(iv) Calculate the mean.


Find the probability that the student scored
(i) 1 mark,

> Answer(c)(i)
(ii) 4 marks,

Answer(c)(ii)
(iii) fewer than 6 marks.

3 The colours of 30 cars in a car park are shown in the frequency table.

| Colour | Frequency |
| :---: | :---: |
| Red | 5 |
| Silver | 15 |
| Black | 6 |
| White | 4 |

(a) Complete the bar chart to represent this information.

(b) Write down the mode.

6 The number of ice-creams sold in a shop each month is shown in the table.

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> ice-creams <br> sold | 1300 | 1200 | 1700 | 1800 | 2300 | 2500 | 2800 | 2600 | 1500 | 1600 | 1100 | 1900 |

(a) (i) Find the range.
Answer(a)(i)
(ii) Calculate the mean.

Answer(a)(ii)
(iii) Find the median.
Answer(a)(iii)
(b) The numbers of chocolate, strawberry and vanilla ice-creams sold are shown in the table.

| Flavour | Number of ice-creams | Pie chart sector angle |
| :---: | :---: | :---: |
| Chocolate | 4200 | $140^{\circ}$ |
| Strawberry | 3600 |  |
| Vanilla | 3000 |  |

(i) Complete the table by working out the sector angles for strawberry and vanilla.
(ii) Complete the pie chart below and label the sectors.

(c) The table shows the average temperature and the number of ice-creams sold each month.

| Month | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Temperature <br> $\left({ }^{\circ} \mathrm{C}\right)$ | 5.6 | 5.7 | 7.0 | 11.4 | 16.0 | 23.3 | 23.4 | 20.0 | 15.5 | 11.5 | 8.0 | 14.0 |
| Number of <br> ice-creams <br> sold | 1300 | 1200 | 1700 | 1800 | 2300 | 2500 | 2800 | 2600 | 1500 | 1600 | 1100 | 1900 |

(i) Complete the scatter diagram for the months August to December.

The points for January to July are plotted for you.

Number of ice-creams sold

(ii) What type of correlation does the scatter diagram show?
Answer(c)(ii)
(iii) Write down a statement connecting the number of ice-creams sold to the average monthly temperature.

Answer(c)(iii)

3288 students took part in a quiz.
There were three questions in the quiz.
Each correct answer scored 1 point.
The pie chart shows the results.

(a) Find the value of $t$.
(b) Find the number of students who scored 2 points.
(c) Find the modal number of points.
(d) (i) Use the information in the pie chart to complete the frequency table for the 288 students.

| Number of points | 0 | 1 | 2 | 3 |
| :---: | :--- | :--- | :--- | :--- |
| Number of students |  |  |  |  |

(ii) Calculate the mean number of points.
[3]
(e) One student is chosen at random.

Find the probability that this student scored
(i) 3 points,
(ii) at least 1 point,

(iii) more than 3 points.

> Answer(e)(iii)
(f) 1440 students took part in the same quiz.

How many students would be expected to score 3 points?

8 The table below shows the marks scored by a group of students in a test.

| Mark | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 10 | 8 | 16 | 11 | 7 | 8 | 6 | 9 |

(a) Find the mean, median and mode.

## Answer(a) mean

(b) The table below shows the time ( $t$ minutes) taken by the students, to complete the test.

| Time $(t)$ | $0<t \leqslant 10$ | $10<t \leqslant 20$ | $20<t \leqslant 30$ | $30<t \leqslant 40$ | $40<t \leqslant 50$ | $50<t \leqslant 60$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 2 | 19 | 16 | 14 | 15 | 9 |

(i) Cara rearranges this information into a new table.

Complete her table.

| Time $(t)$ | $0<t \leqslant 20$ | $20<t \leqslant 40$ | $40<t \leqslant 50$ | $50<t \leqslant 60$ |
| :--- | :---: | :---: | :---: | :---: |
| Frequency |  |  |  | 9 |

(ii) Cara wants to draw a histogram to show the information in part (b)(i).

Complete the table below to show the interval widths and the frequency densities.

|  | $0<t \leqslant 20$ | $20<t \leqslant 40$ | $40<t \leqslant 50$ | $50<t \leqslant 60$ |
| :--- | :---: | :---: | :---: | :---: |
| Interval <br> width |  |  |  | 10 |
| Frequency <br> density |  |  |  | 0.9 |

(c) Some of the students were asked how much time they spent revising for the test.

10 students revised for 2.5 hours, 12 students revised for 3 hours and $n$ students revised for 4 hours.

The mean time that these students spent revising was 3.1 hours.
Find $n$.
Show all your working.


6

| Time <br> $(t$ mins $)$ | $0<t \leqslant 20$ | $20<t \leqslant 35$ | $35<t \leqslant 45$ | $45<t \leqslant 55$ | $55<t \leqslant 70$ | $70<t \leqslant 80$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 6 | 15 | 19 | 37 | 53 | 20 |

The table shows the times taken, in minutes, by 150 students to complete their homework on one day.
(a) (i) In which interval is the median time?
Answer(a)(i)
(ii) Using the mid-interval values $10,27.5, \ldots \ldots .$. .calculate an estimate of the mean time.

(b) (i) Complete the table of cumulative frequencies.

| Time <br> $(t$ mins $)$ | $t \leqslant 20$ | $t \leqslant 35$ | $t \leqslant 45$ | $t \leqslant 55$ | $t \leqslant 70$ | $t \leqslant 80$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cumulative <br> frequency | 6 | 21 |  |  |  |  |

(ii) On the grid, label the horizontal axis from 0 to 80 , using the scale 1 cm represents 5 minutes and the vertical axis from 0 to 150 , using the scale 1 cm represents 10 students.

Draw a cumulative frequency diagram to show this information.

(i) the median time,

Answer(c)(i)
 $\min$
(ii) the inter-quartile range,

$$
\text { Answer(c)(ii) ......................................... } \min
$$

(iii) the number of students whose time was in the range $50<\mathrm{t} \leqslant 60$,
Answer(c)(iii)
(iv) the probability, as a fraction, that a student, chosen at random, took longer than 50 minutes,
Answer(c)(iv)
(v) the probability, as a fraction, that two students, chosen at random, both took longer than 50 minutes.
Answer(c)(v)


The masses of 200 parcels are recorded.
The results are shown in the cumulative frequency diagramabove.
(a) Find
(i) the median,

> Answer(a)(i)
kg [1]
(ii) the lower quartile,
Answer(a)(ii)
(iii) the inter-quartile range,

Answer(a)(iii)
kg [1]
(iv) the number of parcels with a mass greater than 3.5 kg .
Answer(a)(iv)
(b) (i) Use the information from the cumulative frequency diagram to complete the grouped frequency table.

| Mass ( $m$ ) kg | $0<m \leqslant 4$ | $4<m \leqslant 6$ | $6<m \leqslant 7$ | $7<m \leqslant 10$ |
| :---: | :---: | :---: | :---: | :---: |
| Frequency | 36 |  |  | 50 |

(ii) Use the grouped frequency table to calculate an estimate of the mean.
(iii) Complete the frequency density table and use it to complete the histogram.


7 (a) A group of students sat an examination. Each student got one of the grades $A, B, C$ or $D$. The pie chart shows these results.


## NOT TO SCALE

36 students got grade A, shown by an angle of $108^{\circ}$.
(i) Calculate the total number of students who sat the examination.
(ii) How many students did not get grade $A$ ?
(iii) The ratio of the number of students getting grades $B, C$ or $D$ is $4: 5 \odot^{3}$.

Find the number of students getting each grade.
(iv) Work out the angles in the pie chart for grades $B, C$ and $D$.
(v) Find the ratio, in its lowest terms,
the number of students with grade $A$ : the number of students with grade $B$.
(b) A group of children were asked how much money they had saved. The histogram and table show the results.

Frequency
Density


| Money saved $(\$ m)$ | $0<m \leqslant 20$ | $20<m \leqslant 30$ | $30<m \leqslant 40$ | $40<m \leqslant 70$ |
| :---: | :---: | :---: | :---: | :---: |
| Frequency | 25 | $p$ | $q$ | $r$ |

Use the histogram to calculate the values of $p, q$ and $r$.

## 8 Answer the whole of this question on a sheet of graph paper.

120 passengers on an aircraft had their baggage weighed. The results are shown in the table.

| Mass of baggage $(M \mathrm{~kg})$ | $0<M \leqslant 10$ | $10<M \leqslant 15$ | $15<M \leqslant 20$ | $20<M \leqslant 25$ | $25<M \leqslant 40$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of passengers | 12 | 32 | 28 | 24 | 24 |

(a) (i) Write down the modal class.
(ii) Calculate an estimate of the mean mass of baggage for the 120 passengers. Show all your working.
(iii) Sophia draws a pie chart to show the data.

What angle should she have in the $0<M \leqslant 10$ sector?
(b) Using a scale of 2 cm to represent 5 kg , draw a horizontal axis for $0<M \leqslant 40$.

Using an area scale of $1 \mathrm{~cm}^{2}$ to represent 1 passenger, draw a histogram for this data.

6 (a) Students are given marks $0,1,2,3$ or 4 for a piece of work.
The table shows the number of students getting each mark.

| Mark | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 3 | 10 | 12 | 9 | $x$ |

(i) The mean mark is 2.125 .

Find the value of $x$.
(ii) Write down the lower quartile mark.
(b) The heights ( $h$ centimetres) of flowers in a shop are shown in the histogram below.

All the flowers are less than 60 cm high.
One bar has not been drawn on the histogram.

(i) There are 25 flowers in the interval $20<h \leqslant 25$.

How many flowers are there in the intervals
(a) $25<h \leqslant 30$,
(b) $10<h \leqslant 20$ ?
(ii) There are 42 flowers in the interval $30<h \leqslant 60$.

This can be shown by a single bar on the histogram.
Calculate the height of this bar.
(iii) Calculate an estimate of the mean height of the flowers.

## Answer the whole of this question on one sheet of graph paper.

The heights $(h \mathrm{~cm})$ of 270 students in a school are measured and the results are shown in the table.

| $h$ | Frequency |
| :---: | :---: |
| $120<h \leqslant 130$ | 15 |
| $130<h \leqslant 140$ | 24 |
| $140<h \leqslant 150$ | 36 |
| $150<h \leqslant 160$ | 45 |
| $160<h \leqslant 170$ | 50 |
| $170<h \leqslant 180$ | 43 |
| $180<h \leqslant 190$ | 37 |
| $190<h \leqslant 200$ | 20 |

(a) Write down the modal group.
(b) (i) Calculate an estimate of the mean height.
(ii) Explain why the answer to part (b)(i) is an estimate.
(c) The following table shows the cumulative frequencies for the heights of the students.

| $h$ | Cumulative frequency |
| :---: | :---: |
| $h \leqslant 120$ | 0 |
| $h \leqslant 130$ | $p$ |
| $h \leqslant 140$ | $q$ |
| $h \leqslant 150$ | $R$ |
| $h \leqslant 160$ | 120 |
| $h \leqslant 170$ | 170 |
| $h \leqslant 180$ | 213 |
| $h \leqslant 190$ | 250 |
| $h \leqslant 200$ | 270 |

Write down the values of $p, q$ and $r$.
(d) Using a scale of 1 cm to 5 units, draw a horizontal $h$-axis, starting at $h=120$.

Using a scale of 1 cm to 20 units on the vertical axis, draw a cumulative frequency diagram.
(e) Use your diagram to find
(i) the median height,
(ii) the upper quartile,
(iii) the inter-quartile range,
(iv) the 60th percentile.
(f) All the players in the school's basketball team are chosen from the 30 tallest students.

Use your diagram to find the least possible height of any player in the basketball team.

[^0]7 (a) The quiz scores of a class of $n$ students are shown in the table.

| Quiz score | 6 | 7 | 8 | 9 |
| :--- | :---: | :---: | :---: | :---: |
| Frequency (number of students) | 9 | 3 | $a$ | 5 |

The mean score is 7.2 . Find
(i) $a$,
(ii) $n$,
(iii) the median score.
(b) 200 students take a mathematics test.

The cumulative frequency diagram shows the results.


Write down
(i) the median mark,
(ii) the lower quartile,
(iii) the upper quartile,
(iv) the inter-quartile range,
(v) the lowest possible mark scored by the top 40 students,
(vi) the number of students scoring more than 25 marks.
(c) Another group of students takes an English test.

The results are shown in the histogram.


100 students score marks in the range $50<x \leqslant 75$.
(i) How many students score marks in the range $0<x \leqslant 50$ ?
(ii) How many students score marks in the range $75<x \leqslant 100$ ?
(iii) Calculate an estimate of the mean mark of this group of students.

8 (a) The surface area, $A$, of a cylinder, radius $r$ and height $h$, is given by the formula
(i) Calculate the surface area of a cylinder of radius 5 cm and height 9 cm .
(ii) Make $h$ the subject of the formula.
(iii) A cylinder has a radius of 6 cm and a surface area of $377 \mathrm{~cm}^{2}$.

Calculate the height of this cylinder.
(iv) A cylinder has a surface area of $1200 \mathrm{~cm}^{2}$ and its radius and height are equal. Calculate the radius.
(b) (i) On Monday a shop receives $\$ 60.30$ by selling bottles of water at 45 cents each. How many bottles are sold?
(ii) On Tuesday the shop receives $x$ cents by selling bottles of water at 45 cents each. In terms of $x$, how many bottles are sold?
(iii) On Wednesday the shop receives $(x-75)$ cents by selling bottles of water at 48 cents each. In terms of $x$, how many bottles are sold?
(iv) The number of bottles sold on Tuesday was 7 more than the number of bottles sold on Wednesday.
Write down an equation in $x$ and solve your equation.

19 The mass of each of 200 tea bags was checked by an inspector in a factory. The results are shown by the cumulative frequency curve.

(b) the interquartile range,
(c) the number of tea bags with a mass greater than 3.5 grams.

2 (a)

| Grade | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number of students | 1 | 2 | 4 | 7 | 4 | 8 | 2 |

The table shows the grades gained by 28 students in a history test.
(i) Write down the mode.
(ii) Find the median.
(iii) Calculate the mean.
(iv) Two students are chosen at random.

Calculate the probability that they both gained grade 5 .
(v) From all the students who gained grades 4 or 5 or 6 or 7, two are chosen at random.

Calculate the probability that they both gained grade 5
(vi) Students are chosen at random, one by one, from the original 28, until the student chosen has a grade 5 .

Calculate the probability that this is the third student chosen.
(b) Claude goes to school by bus.

The probability that the bus is late is 0.1 .
If the bus is late, the probability that Claude is late to school is 0.8 .
If the bus is not late, the probability that Claude is late to school is 0.05 .
(i) Calculate the probability that the bus is late and Claude is late to school.
(ii) Calculate the probability that Claude is late to school.
(iii) The school term lasts 56 days.

How many days would Claude expect to be late?

6 (a) Each student in a class is given a bag of sweets.
The students note the number of sweets in their bag.
The results are shown in the table, where $0 \leqslant x<10$.

| Number of sweets | 30 | 31 | 32 |
| :---: | :---: | :---: | :---: |
| Frequency (number of bags) | 10 | 7 | $x$ |

(i) State the mode.
(ii) Find the possible values of the median.
(iii) The mean number of sweets is 30.65 .

Find the value of $x$.
(b) The mass, $m$ grams, of each of 200 chocolates is noted and the resuls are shown in the table.

(i) Calculate an estimate of the mean mass of a chocolate.
(ii) On a histogram, the height of the column for the $20<m \leqslant 22$ interval is 11.5 cm .

Calculate the heights of the other three columns.

## Do not draw the histogram.

20 The number of hours that a group of 80 students spent using a computer in a week was recorded. The results are shown by the cumulative frequency curve.


Use the cumulative frequency curve to find
(a) the median,
(b) the upper quartile,

> Answer(b)
h [1]
(c) the interquartile range,

Answer(c)
(d) the number of students who spent more than 50 hours using a computer in a week.
Answer(d)

8 Fifty students are timed when running one kilometre.
The results are shown in the table.

| Time <br> $(t$ minutes $)$ | $4.0<t \leqslant 4.5$ | $4.5<t \leqslant 5.0$ | $5.0<t \leqslant 5.5$ | $5.5<t \leqslant 6.0$ | $6.0<t \leqslant 6.5$ | $6.5<t \leqslant 7.0$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 2 | 7 | 8 | 18 | 10 | 5 |

(a) Write down the modal time interval.
(b) Calculate an estimate of the mean time.

Answer(b)
$\min [4]$
(c) A new frequency table is made from the results shown in the table above.

| Time <br> $(t$ minutes $)$ | $4.0<t \leqslant 5.5$ | $5.5<t \leqslant 6.0$ | $6.0<t \leqslant 7.0$ |
| :--- | :---: | :---: | :---: |
| Frequency |  | 18 |  |

(i) Complete the table by filling in the two empty boxes.
(ii) On the grid below, complete an accurate histogram to show the information in this new table.

(iii) Find the number of students represented by $1 \mathrm{~cm}^{2}$ on the histogram.

5 The cumulative frequency table shows the distribution of heights, $h$ centimetres, of 200 students.

| Height $(h \mathrm{~cm})$ | $\leqslant 130$ | $\leqslant 140$ | $\leqslant 150$ | $\leqslant 160$ | $\leqslant 165$ | $\leqslant 170$ | $\leqslant 180$ | $\leqslant 190$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Cumulative frequency | 0 | 10 | 50 | 95 | 115 | 145 | 180 | 200 |

(a) Draw a cumulative frequency diagram to show the information in the table.

(b) Use your diagram to find
(i) the median,
Answer(b)(i) ......................................... cm [1]
(ii) the upper quartile,

Answer(b)(ii)
(iii) the interquartile range.

Answer(b)(iii)
cm [1]
(c) (i) One of the 200 students is chosen at random.

Use the table to find the probability that the height of this student is greater than 170 cm . Give your answer as a fraction.
(ii) One of the 200 students is chosen at random and then a second student is chosen at random from the remaining students.

Calculate the probability that one has a height greater than 170 cm and the other has a height of 140 cm or less. Give your answer as a fraction.
Answer(c)(ii)
(d) (i) Complete this frequency table which shows the distribution of the heights of the 200 students.

| Height $(h \mathrm{~cm})$ | $130<h \leqslant 140$ | $140<h \leqslant 150$ | $150<h \leqslant 160$ | $160<h \leqslant 165$ | $165<h \leqslant 170$ | $170<h \leqslant 180$ | $180<h \leqslant 190$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 10 | 40 | 45 | 20 |  |  |  |

(ii) Complete this histogram to show the distribution of the heights of the 200 students.


380 boys each had their mass, $m$ kilograms, recorded.
The cumulative frequency diagram shows the results.


Answer(a)(ii) $\qquad$ kg
(iii) the interquartile range.

Answer(a)(iii) kg
(b) How many boys had a mass greater than 60 kg ?
(c) (i) Use the cumulative frequency graph to complete this frequency table.

| Mass, $m$ | Frequency |
| :---: | :---: |
| $30<m \leqslant 40$ | 8 |
| $40<m \leqslant 50$ |  |
| $50<m \leqslant 60$ | 14 |
| $60<m \leqslant 70$ | 22 |
| $70<m \leqslant 80$ |  |
| $80<m \leqslant 90$ | 10 |

(ii) Calculate an estimate of the mean mass.
(ii) On the grid, complete the histogram to show the information in the table.


13


The cumulative frequency diagram shows the height of plants measured in an experiment. From the diagram, estimate
(a) (i) the lower quartile,

Answer (a)(i)
(ii) the inter-quartile range,

Answer (a)(iii) .cm
(b) the number of plants with a height greater than 25 cm .

Answer (b)

14 For a holiday in 1998, Stefan wanted to change 250 Cypriot pounds ( $\mathfrak{f}$ ) into Greek Drachma.
He first had to pay a bank charge of $1 \frac{1}{2} \%$ of the $£ 250$.
He then changed the remaining pounds into Drachma at a rate of $£ 1=485$ Drachma.
Calculate how many Drachma Stefan received, giving your answer to the nearest 10 .
$\qquad$


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