

## 6. List of formulae and tables of the normal distribution (MF9)

### PURE MATHEMATICS

#### Algebra

For the quadratic equation  $ax^2 + bx + c = 0$ :

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

For an arithmetic series:

$$u_n = a + (n-1)d, \quad S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

For a geometric series:

$$u_n = ar^{n-1}, \quad S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1), \quad S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + b^n, \text{ where } n \text{ is a positive integer}$$

$$\text{and } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 \dots, \text{ where } n \text{ is rational and } |x| < 1$$

#### Trigonometry

Arc length of circle =  $r\theta$  ( $\theta$  in radians)

Area of sector of circle =  $\frac{1}{2}r^2\theta$  ( $\theta$  in radians)

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$$

$$\cos^2 \theta + \sin^2 \theta \equiv 1, \quad 1 + \tan^2 \theta \equiv \sec^2 \theta, \quad \cot^2 \theta + 1 \equiv \operatorname{cosec}^2 \theta$$

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2 \cos^2 A - 1 \equiv 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Principal values:

$$-\frac{1}{2}\pi \leq \sin^{-1} x \leq \frac{1}{2}\pi$$

$$0 \leq \cos^{-1} x \leq \pi$$

$$-\frac{1}{2}\pi < \tan^{-1} x < \frac{1}{2}\pi$$

*Differentiation*

$f(x)$	$f'(x)$
$x^n$	$nx^{n-1}$
$\ln x$	$\frac{1}{x}$
$e^x$	$e^x$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$uv$	$u \frac{dv}{dx} + v \frac{du}{dx}$
$\frac{u}{v}$	$\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

If  $x = f(t)$  and  $y = g(t)$  then  $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

*Integration*

$f(x)$	$\int f(x) dx$
$x^n$	$\frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$
$\frac{1}{x}$	$\ln x  + c$
$e^x$	$e^x + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$
$\sec^2 x$	$\tan x + c$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

*Vectors*

If  $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$  and  $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$  then

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}| |\mathbf{b}| \cos \theta$$

*Numerical integration*

Trapezium rule:

$$\int_a^b f(x) dx \approx \frac{1}{2}h\{y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n\}, \text{ where } h = \frac{b-a}{n}$$

**MECHANICS***Uniformly accelerated motion*

$$v = u + at, \quad s = \frac{1}{2}(u + v)t, \quad s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

*Motion of a projectile*

Equation of trajectory is:

$$y = x \tan \theta - \frac{gx^2}{2V^2 \cos^2 \theta}$$

*Elastic strings and springs*

$$T = \frac{\lambda x}{l}, \quad E = \frac{\lambda x^2}{2l}$$

*Motion in a circle*

For uniform circular motion, the acceleration is directed towards the centre and has magnitude

$$\omega^2 r \quad \text{or} \quad \frac{v^2}{r}$$

*Centres of mass of uniform bodies*Triangular lamina:  $\frac{2}{3}$  along median from vertexSolid hemisphere of radius  $r$ :  $\frac{3}{8}r$  from centreHemispherical shell of radius  $r$ :  $\frac{1}{2}r$  from centreCircular arc of radius  $r$  and angle  $2\alpha$ :  $\frac{r \sin \alpha}{\alpha}$  from centreCircular sector of radius  $r$  and angle  $2\alpha$ :  $\frac{2r \sin \alpha}{3\alpha}$  from centreSolid cone or pyramid of height  $h$ :  $\frac{3}{4}h$  from vertex

## PROBABILITY AND STATISTICS

### Summary statistics

For ungrouped data:

$$\bar{x} = \frac{\sum x}{n}, \quad \text{standard deviation} = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

For grouped data:

$$\bar{x} = \frac{\sum xf}{\sum f}, \quad \text{standard deviation} = \sqrt{\frac{\sum (x - \bar{x})^2 f}{\sum f}} = \sqrt{\frac{\sum x^2 f}{\sum f} - \bar{x}^2}$$

### Discrete random variables

$$E(X) = \sum xp$$

$$\text{Var}(X) = \sum x^2 p - \{E(X)\}^2$$

For the binomial distribution  $B(n, p)$ :

$$p_r = \binom{n}{r} p^r (1-p)^{n-r}, \quad \mu = np, \quad \sigma^2 = np(1-p)$$

For the Poisson distribution  $Po(a)$ :

$$p_r = e^{-a} \frac{a^r}{r!}, \quad \mu = a, \quad \sigma^2 = a$$

### Continuous random variables

$$E(X) = \int xf(x) dx$$

$$\text{Var}(X) = \int x^2 f(x) dx - \{E(X)\}^2$$

### Sampling and testing

Unbiased estimators:

$$\bar{x} = \frac{\sum x}{n}, \quad s^2 = \frac{1}{n-1} \left( \sum x^2 - \frac{(\sum x)^2}{n} \right)$$

Central Limit Theorem:

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Approximate distribution of sample proportion:

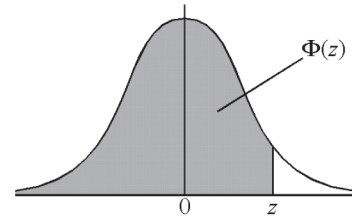
$$N\left(p, \frac{p(1-p)}{n}\right)$$

**THE NORMAL DISTRIBUTION FUNCTION**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $z$ , the table gives the value of  $\Phi(z)$ , where

$$\Phi(z) = P(Z \leq z).$$

For negative values of  $z$  use  $\Phi(-z) = 1 - \Phi(z)$ .



z											ADD								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

**Critical values for the normal distribution**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that

$$P(Z \leq z) = p.$$

$p$	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$z$	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291