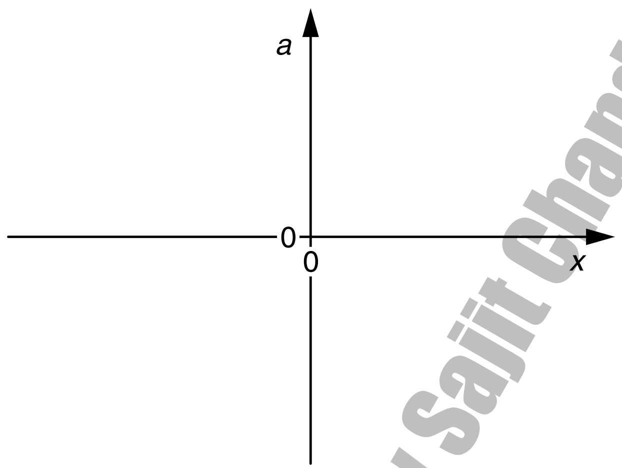
1 **(a)** **(i)** Define simple harmonic motion.

....

....

....

(ii) On the axes of Fig. 4.1, sketch the variation with displacement x of the acceleration a of a particle undergoing simple harmonic motion.



[4]

Fig. 4.1

(b) A strip of metal is clamped to the edge of a bench and a mass is hung from its free end as shown in Fig. 4.2.

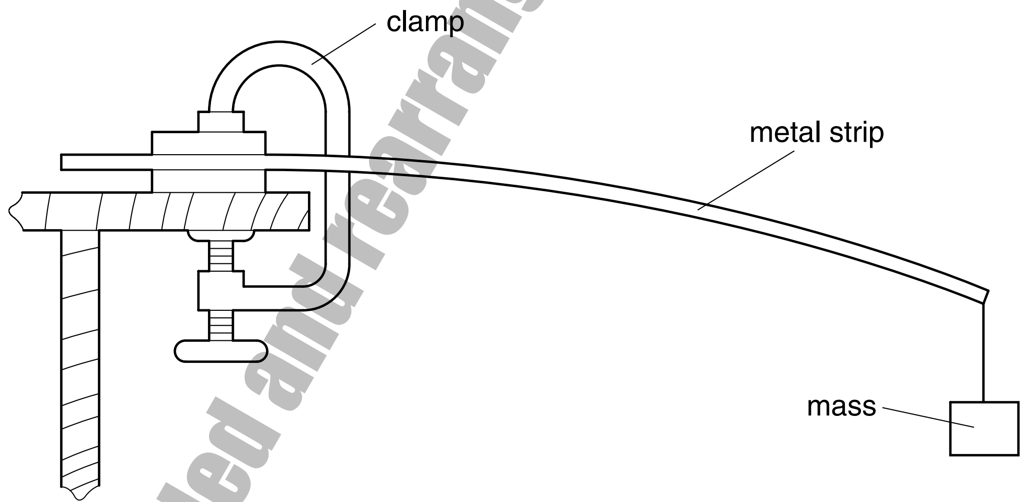


Fig. 4.2

The end of the strip is pulled downwards and then released. Fig. 4.3 shows the variation with time t of the displacement y of the end of the strip.

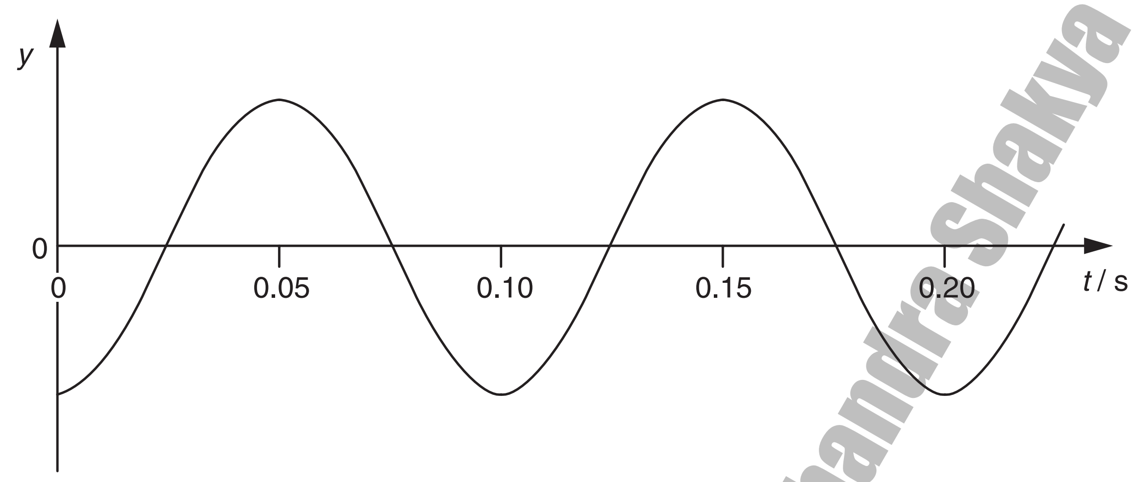


Fig. 4.3

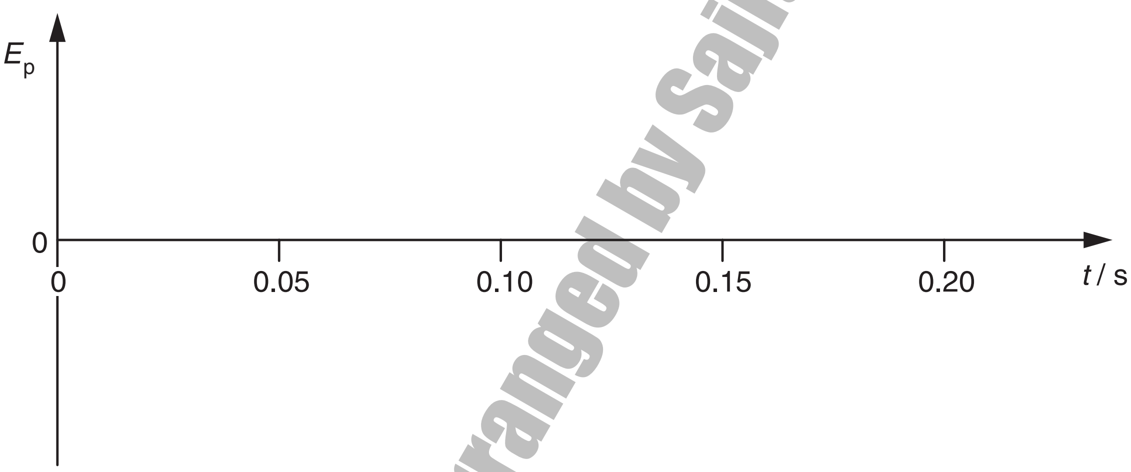


Fig. 4.4

On Fig. 4.4, show the corresponding variation with time t of the potential energy E_p of the vibrating system. [3]

(c) The string supporting the mass breaks when the end of the strip is at its lowest point in an oscillation. Suggest what change, if any, will occur in the period and amplitude of the subsequent motion of the end of the strip.

period: ....

amplitude: ....[2]

For Examiner's Use

2 **(a)** The defining equation of simple harmonic motion is

$$a = -\omega^2 x.$$

(i) Identify the symbols in the equation.

a ....

ω ....

x ....

(ii) State the significance of the negative (–) sign in the equation.

....

....

[3]

(b) A frictionless trolley of mass m is held on a horizontal surface by means of two similar springs, each of spring constant k. The springs are attached to fixed points as illustrated in Fig. 2.1.

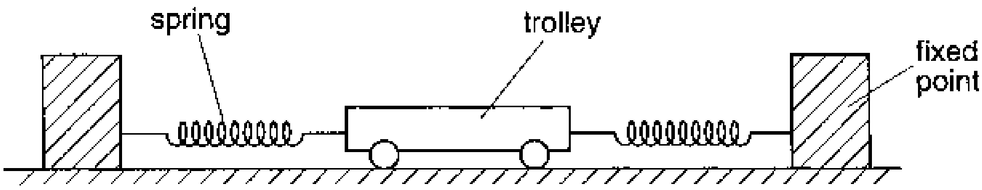


Fig. 2.1

When the trolley is in equilibrium, the extension of each spring is e.

The trolley is then displaced a small distance x to the right along the axis of the springs. Both springs remain extended.

(i) Show that the magnitude F of the restoring force acting on the trolley is given by

$$F = 2kx.$$

[2]

(ii) The trolley is then released. Show that the acceleration a of the trolley is given by

$$a = \frac{-2kx}{m}.$$

[2]

(iii) The mass m of the trolley is 900 g and the spring constant k is 120 N m^{-1}. By comparing your answer to **(a)(i)** and the equation in **(b)(ii)**, determine the frequency of oscillation of the trolley.

frequency = ... Hz [3]

(c) Suggest why the trolley in **(b)** provides a simple model for the motion of an atom in a crystal.

....

....

.... [2]

3 A vertical spring supports a mass, as shown in Fig. 4.1.

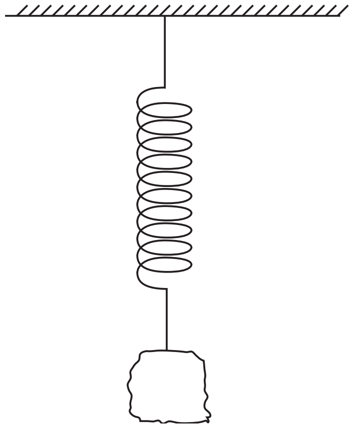


Fig. 4.1

The mass is displaced vertically then released. The variation with time *t* of the displacement *y* from its mean position is shown in Fig. 4.2.

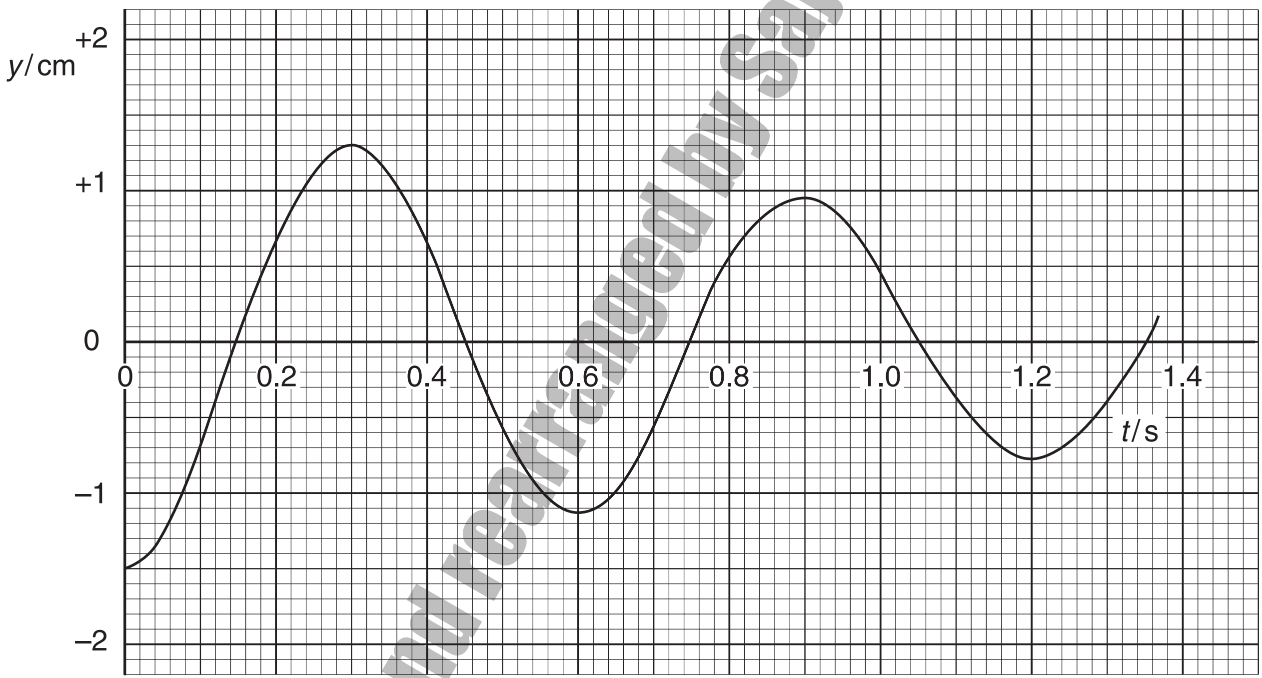


Fig. 4.2

A student claims that the motion of the mass may be represented by the equation

$$y = y_0 \sin \omega t.$$

(a) Give two reasons why the use of this equation is inappropriate.

1. ....

....

2. ....

.... [2]

(b) Determine the angular frequency ω of the oscillations.

angular frequency = ... rad s^{-1} [2]

(c) The mass is a lump of plasticine. The plasticine is now flattened so that its surface area is increased. The mass of the lump remains constant and the large surface area is horizontal.
The plasticine is displaced downwards by 1.5 cm and then released.
On Fig. 4.2, sketch a graph to show the subsequent oscillations of the plasticine. [3]

4 A tube, closed at one end, has a constant area of cross-section A. Some lead shot is placed in the tube so that the tube floats vertically in a liquid of density ρ, as shown in Fig. 4.1.

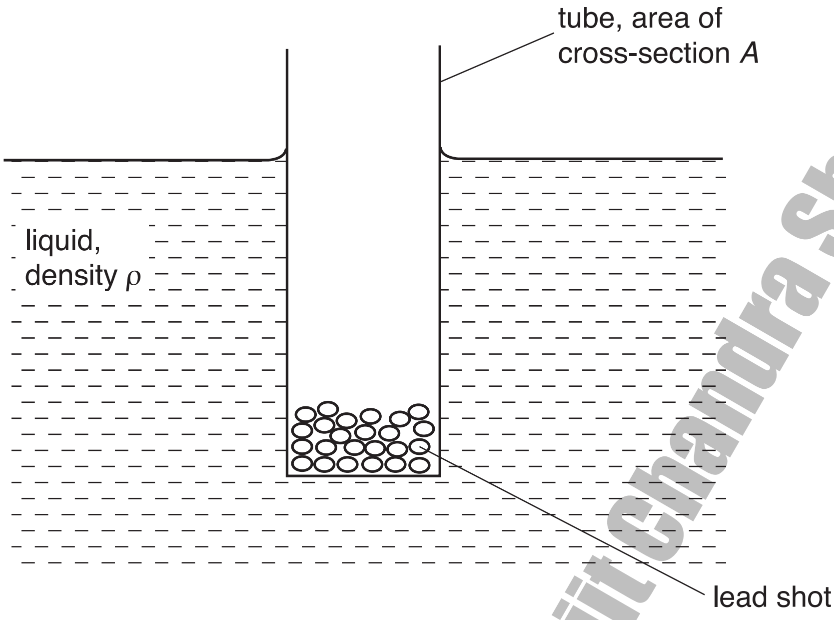


Fig. 4.1

The total mass of the tube and its contents is M.
When the tube is given a small vertical displacement and then released, the vertical acceleration a of the tube is related to its vertical displacement y by the expression

$$a = -\frac{A\rho g}{M}y,$$

where g is the acceleration of free fall.

(a) Define *simple harmonic motion*.

....

....

....[2]

(b) Show that the tube is performing simple harmonic motion with a frequency f given by

$$f = \frac{1}{2\pi}\sqrt{\frac{A\rho g}{M}}.$$

[3]

(c) Fig. 4.2 shows the variation with time t of the vertical displacement y of the tube in another liquid.

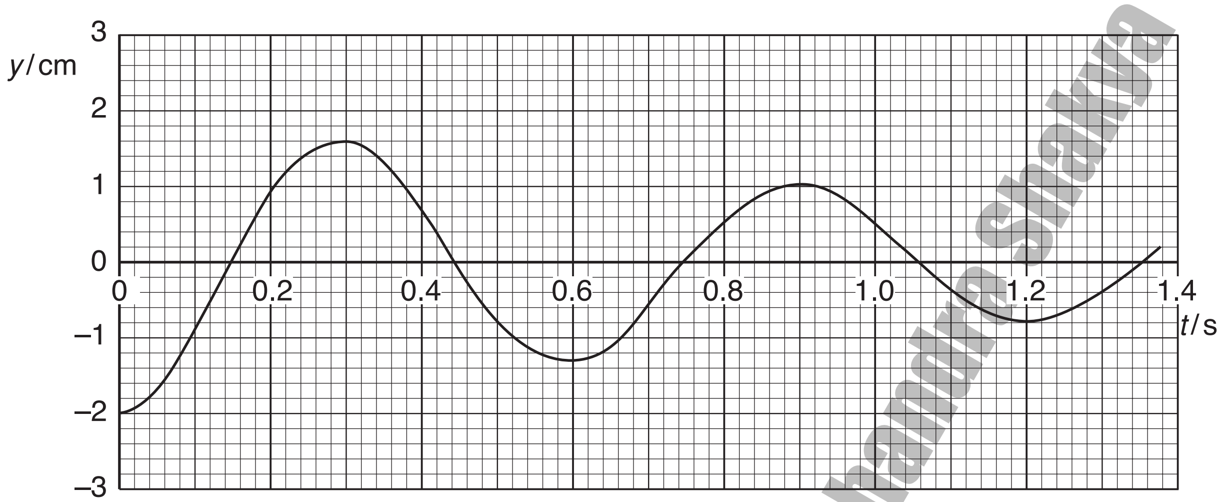


Fig. 4.2

(i) The tube has an external diameter of 2.4 cm and is floating in a liquid of density 950 kg m^{-3}. Assuming the equation in **(b)**, calculate the mass of the tube and its contents.

mass = ... kg [3]

(ii) State what feature of Fig. 4.2 indicates that the oscillations are damped.

....

....[1]

5 A piston moves vertically up and down in a cylinder, as illustrated in Fig. 4.1.

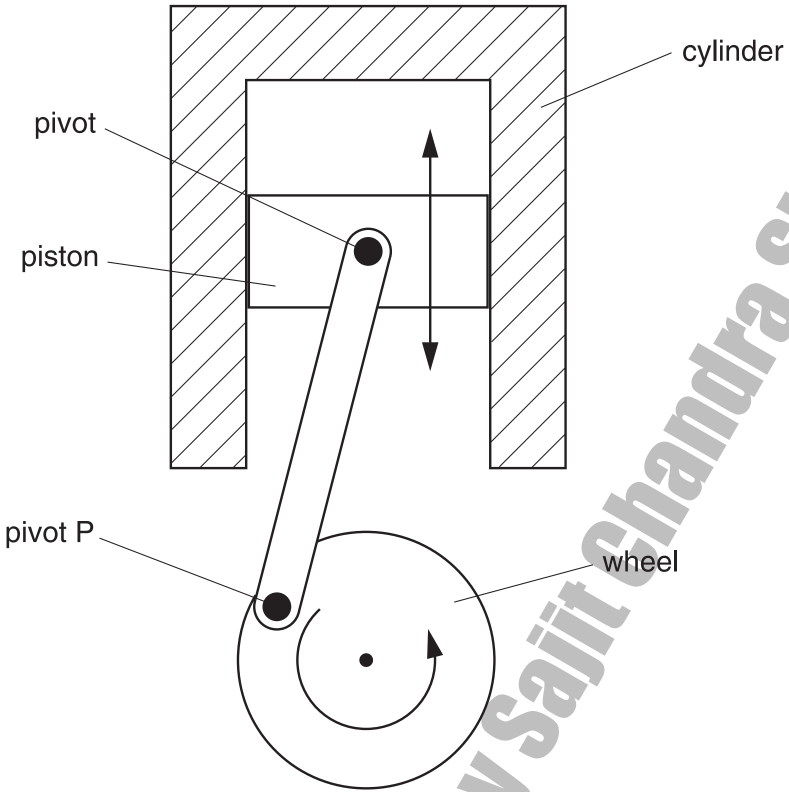


Fig. 4.1

The piston is connected to a wheel by means of a rod that is pivoted at the piston and at the wheel. As the piston moves up and down, the wheel is made to rotate.

(a) (i) State the number of oscillations made by the piston during one complete rotation of the wheel.

number = ... [1]

(ii) The wheel makes 2400 revolutions per minute. Determine the frequency of oscillation of the piston.

frequency = ... Hz [1]

(b) The amplitude of the oscillations of the piston is 42 mm.

Assuming that these oscillations are simple harmonic, calculate the maximum values for the piston of

(i) the linear speed,

speed = ... $m\,s^{-1}$ [2]

(ii) the acceleration.

acceleration = ... $m\,s^{-2}$ [2]

(c) On Fig. 4.1, mark a position of the pivot P for the piston to have

(i) maximum speed (mark this position S), [1]

(ii) maximum acceleration (mark this position A). [1]

6 An aluminium sheet is suspended from an oscillator by means of a spring, as illustrated in Fig. 3.1.

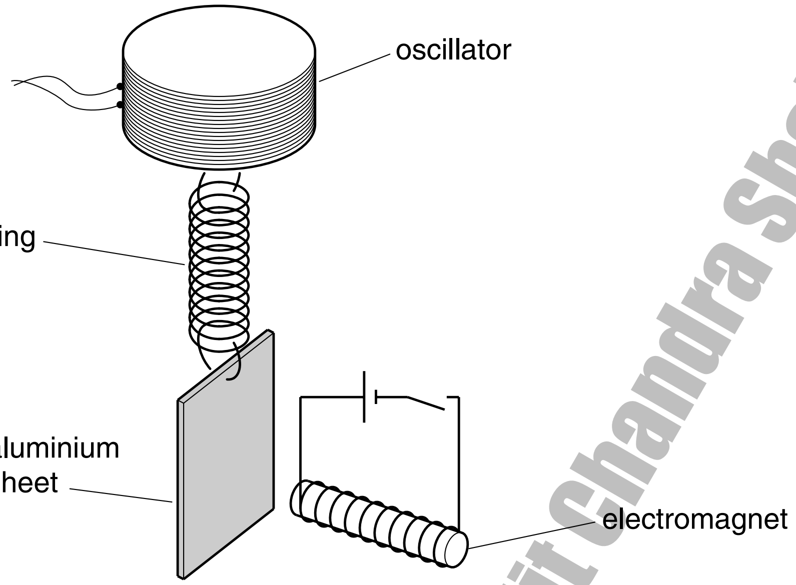


Fig. 3.1

An electromagnet is placed a short distance from the centre of the aluminium sheet.

The electromagnet is switched off and the frequency f of oscillation of the oscillator is gradually increased from a low value. The variation with frequency f of the amplitude a of vibration of the sheet is shown in Fig. 3.2.

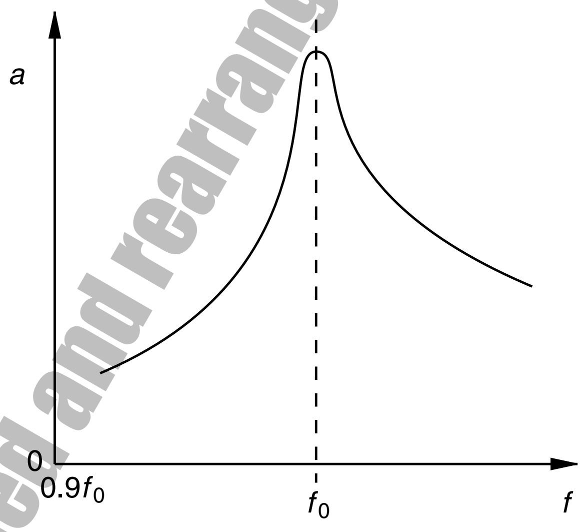


Fig. 3.2

A peak on the graph appears at frequency f_0.

(a) Explain why there is a peak at frequency f_0.

....

....

.... [2]

(b) The electromagnet is now switched on and the frequency of the oscillator is again gradually increased from a low value. On Fig. 3.2, draw a line to show the variation with frequency f of the amplitude a of vibration of the sheet. [3]

(c) The frequency of the oscillator is now maintained at a constant value. The amplitude of vibration is found to decrease when the current in the electromagnet is switched on.

Use the laws of electromagnetic induction to explain this observation.

....

....

....

....

....

.... [4]

7 A tube, closed at one end, has a uniform area of cross-section. The tube contains some sand so that the tube floats upright in a liquid, as shown in Fig. 3.1.

For Examiner's Use

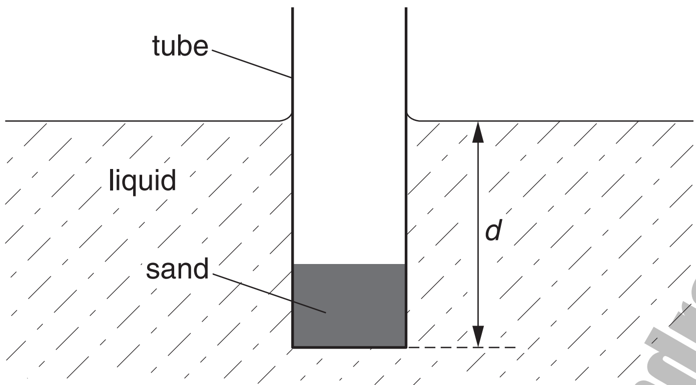


Fig. 3.1

When the tube is at rest, the depth d of immersion of the base of the tube is 16 cm.
The tube is displaced vertically and then released.
The variation with time t of the depth d of the base of the tube is shown in Fig. 3.2.

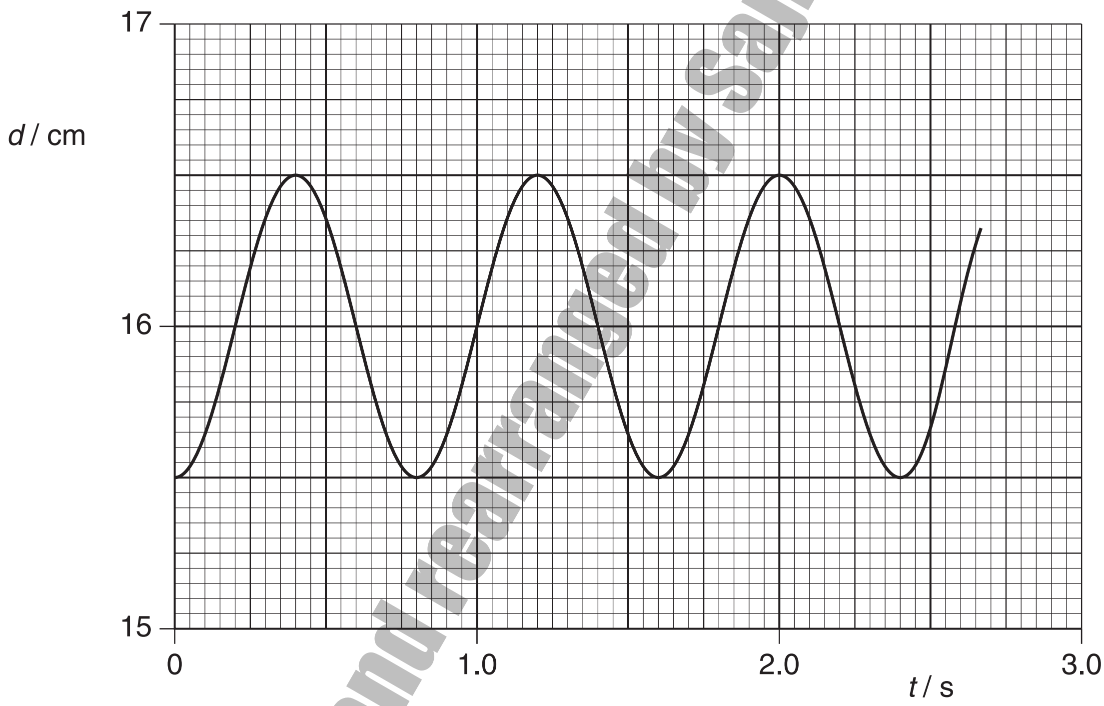


Fig. 3.2

(a) Use Fig. 3.2 to determine, for the oscillations of the tube,

(i) the amplitude,

amplitude = ... cm [1]

(ii) the period.

period = ... s [1]

For Examiner's Use

(b) (i) Calculate the vertical speed of the tube at a point where the depth d is 16.2 cm.

speed = ... cm s^{-1} [3]

(ii) State **one** other depth d where the speed will be equal to that calculated in **(i)**.

d = ... cm [1]

(c) (i) Explain what is meant by *damping*.

....

....

....

.... [2]

(ii) The liquid in **(b)** is now cooled so that, although the density is unchanged, there is friction between the liquid and the tube as it oscillates. Having been displaced, the tube completes approximately 10 oscillations before coming to rest.
On Fig. 3.2, draw a line to show the variation with time t of depth d for the first 2.5 s of the motion. [3]

8 A vertical peg is attached to the edge of a horizontal disc of radius r, as shown in Fig. 4.1.

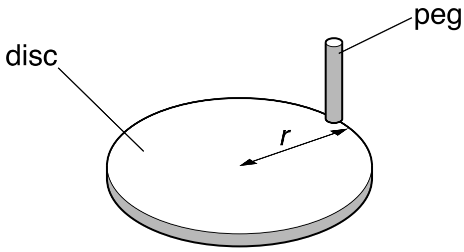


Fig. 4.1

The disc rotates at constant angular speed ω. A horizontal beam of parallel light produces a shadow of the peg on a screen, as shown in Fig. 4.2.

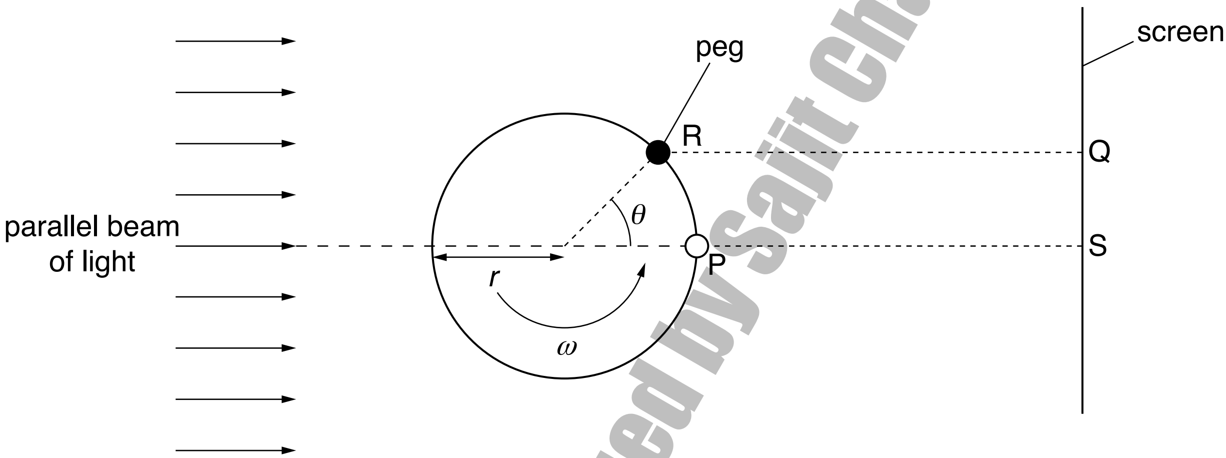


Fig. 4.2 (plan view)

At time zero, the peg is at P, producing a shadow on the screen at S.
At time t, the disc has rotated through angle θ. The peg is now at R, producing a shadow at Q.

(a) Determine,

(i) in terms of ω and t, the angle θ,

.... [1]

(ii) in terms of ω, t and r, the distance SQ.

.... [1]

(b) Use your answer to **(a)(ii)** to show that the shadow on the screen performs simple harmonic motion.

....

....

.... [2]

(c) The disc has radius r of 12 cm and is rotating with angular speed ω of 4.7 rad s^{-1}.

Determine, for the shadow on the screen,

(i) the frequency of oscillation,

frequency = ... Hz [2]

(ii) its maximum speed.

speed = ... cm s^{-1} [2]

9 A student sets out to investigate the oscillation of a mass suspended from the free end of a spring, as illustrated in Fig. 3.1.

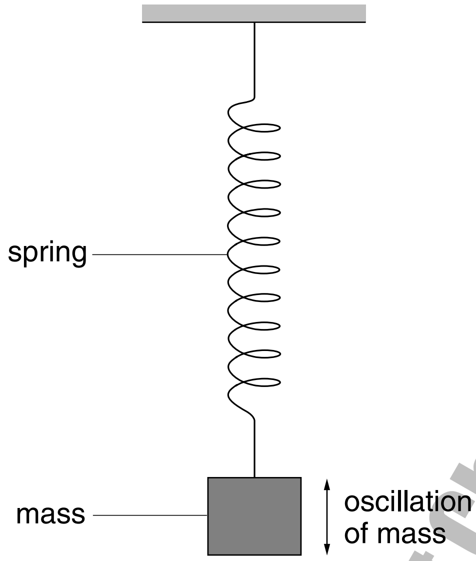


Fig. 3.1

The mass is pulled downwards and then released. The variation with time t of the displacement y of the mass is shown in Fig. 3.2.

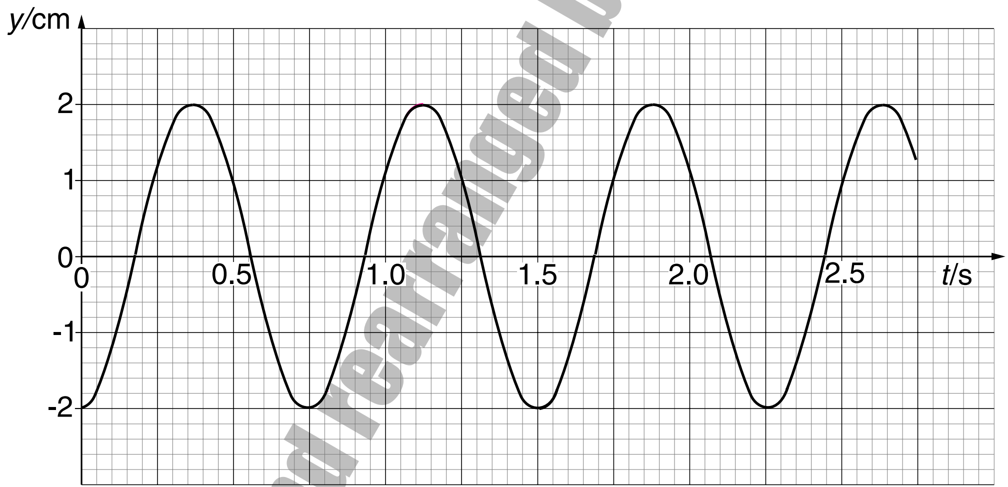


Fig. 3.2

(a) Use information from Fig. 3.2

(i) to explain why the graph suggests that the oscillations are undamped,

....

(ii) to calculate the angular frequency of the oscillations,

angular frequency = ... rad s^{-1}

(iii) to determine the maximum speed of the oscillating mass.

speed = ... m s^{-1}
[6]

(b) **(i)** Determine the resonant frequency f_0 of the mass-spring system.

f_0 = ... Hz

(ii) The student finds that if short impulsive forces of frequency $\frac{1}{2}f_0$ are impressed on the mass-spring system, a large amplitude of oscillation is obtained. Explain this observation.

....

....

....
[3]

10 The vibrations of a mass of 150 g are simple harmonic. Fig. 3.1 shows the variation with displacement x of the kinetic energy E_k of the mass.

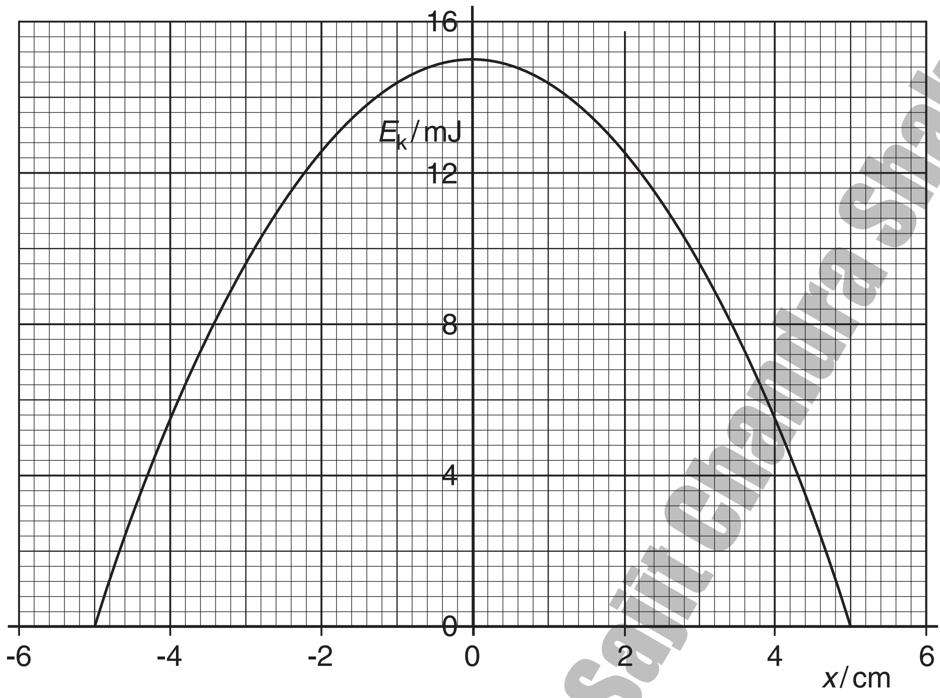


Fig. 3.1

(a) On Fig. 3.1, draw lines to represent the variation with displacement x of

(i) the potential energy of the vibrating mass (label this line P),

(ii) the total energy of the vibrations (label this line T).

[2]

(b) Calculate the angular frequency of the vibrations of the mass.

angular frequency = ... rad s^{-1} [3]

(c) The oscillations are now subject to damping.

(i) Explain what is meant by *damping*.

....

....

....[2]

(ii) The mass loses 20% of its vibrational energy. Use Fig. 3.1 to determine the new amplitude of oscillation. Explain your working.

amplitude = ... cm [2]

11 The centre of the cone of a loudspeaker is oscillating with simple harmonic motion of frequency 1400 Hz and amplitude 0.080 mm.

(a) Calculate, to two significant figures,

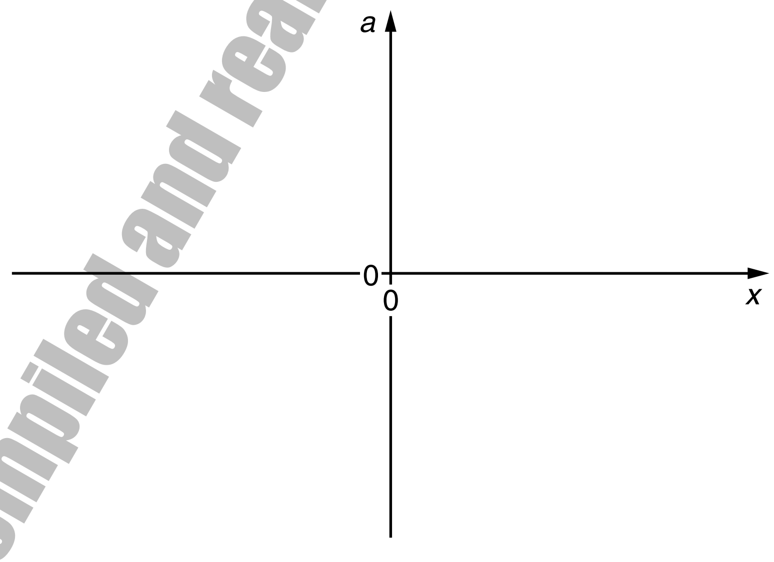
(i) the angular frequency ω of the oscillations,

ω = ………………………………. rad s^{-1} [2]

(ii) the maximum acceleration, in m s^{-2}, of the centre of the cone.

acceleration = ……………………………….. m s^{-2} [2]

(b) On the axes of Fig. 4.1, sketch a graph to show the variation with displacement *x* of the acceleration *a* of the centre of the cone.



[2]

Fig. 4.1

(c) **(i)** State the value of the displacement x at which the speed of the centre of the cone is a maximum.

x = ……………………………… mm [1]

(ii) Calculate, in $m\,s^{-1}$, this maximum speed.

speed = …………………………….. $m\,s^{-1}$ [2]

12 Two vertical springs, each having spring constant k, support a mass. The lower spring is attached to an oscillator as shown in Fig. 3.1.

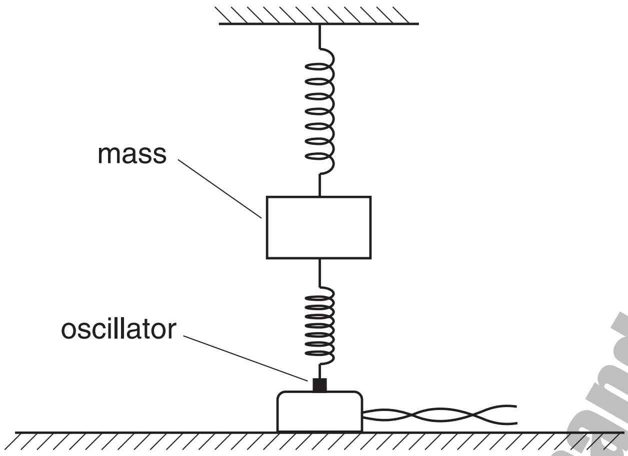


Fig. 3.1

The oscillator is switched off. The mass is displaced vertically and then released so that it vibrates. During these vibrations, the springs are always extended. The vertical acceleration a of the mass m is given by the expression

$$ma = -2kx,$$

where x is the vertical displacement of the mass from its equilibrium position.

(a) Show that, for a mass of 240 g and springs with spring constant 3.0 N cm^{-1}, the frequency of vibration of the mass is approximately 8 Hz.

[4]

(b) The oscillator is switched on and the frequency f of vibration is gradually increased. The amplitude of vibration of the oscillator is constant.

Fig. 3.2 shows the variation with f of the amplitude A of vibration of the mass.

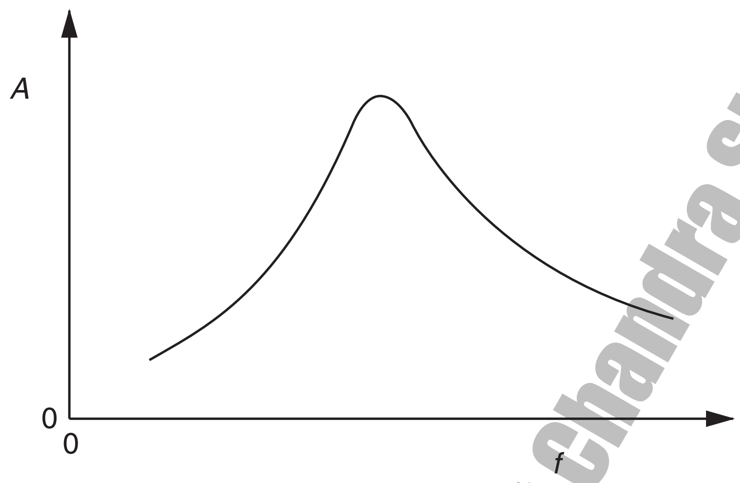


Fig. 3.2

State

(i) the name of the phenomenon illustrated in Fig. 3.2,

.... [1]

(ii) the frequency f_0 at which maximum amplitude occurs.

frequency = ... Hz [1]

(c) Suggest and explain how the apparatus in Fig. 3.1 could be modified to make the peak on Fig. 3.2 flatter, without significantly changing the frequency f_0 at which the peak occurs.

....

....

....

.... [3]

13 A spring is hung from a fixed point. A mass of 130 g is hung from the free end of the spring, as shown in Fig. 3.1.

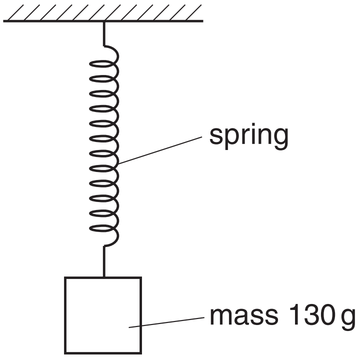


Fig. 3.1

The mass is pulled downwards from its equilibrium position through a small distance *d* and is released. The mass undergoes simple harmonic motion.
Fig. 3.2 shows the variation with displacement *x* from the equilibrium position of the kinetic energy of the mass.

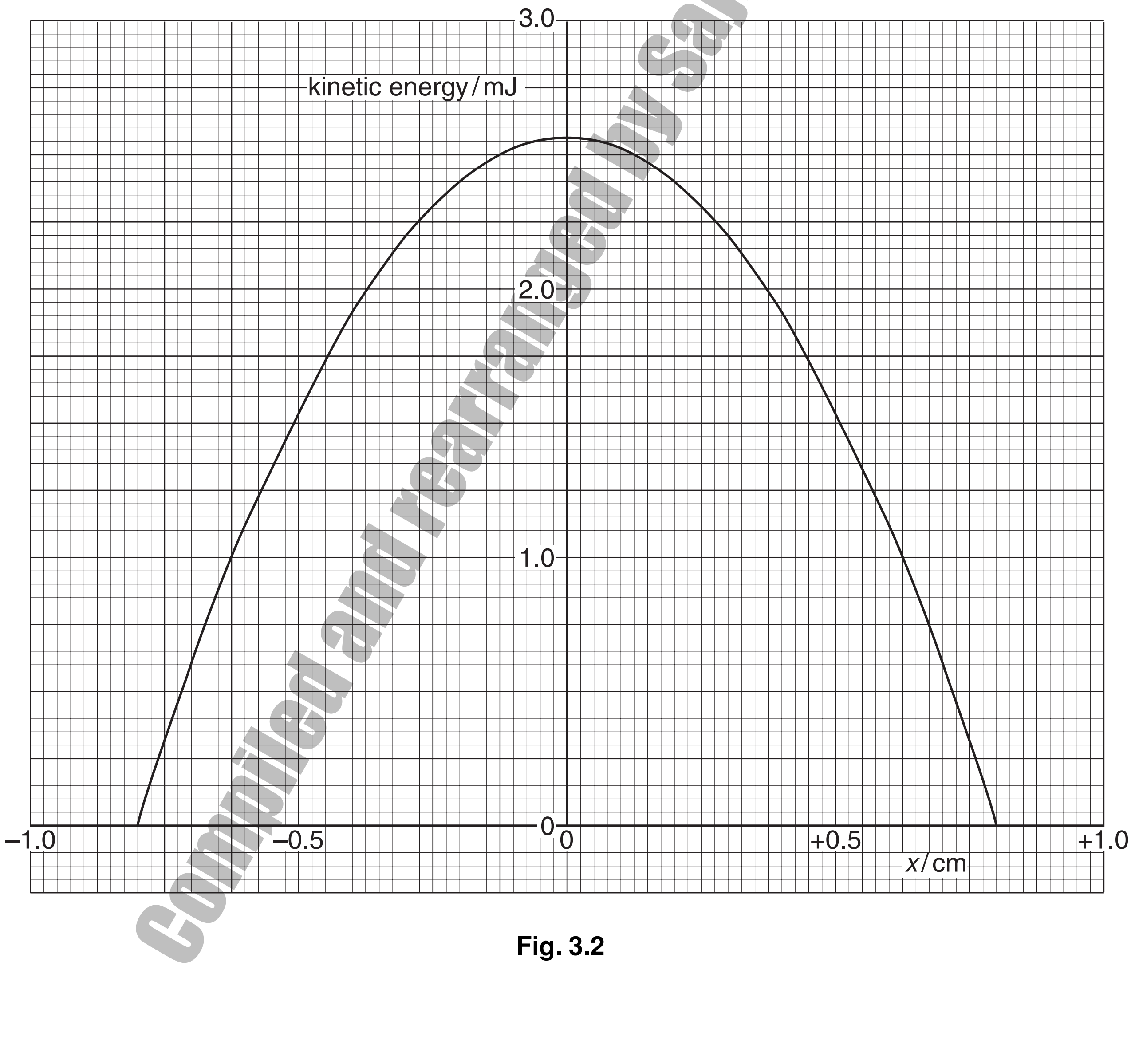


Fig. 3.2

(a) Use Fig. 3.2 to

(i) determine the distance d through which the mass was displaced initially,

d = ... cm [1]

(ii) show that the frequency of oscillation of the mass is approximately 4.0 Hz.

[6]

(b) (i) On Fig. 3.2, draw a line to represent the total energy of the oscillating mass. [1]

(ii) After many oscillations, damping reduces the total energy of the mass to 1.0 mJ. For the oscillations with reduced energy,

1. state the frequency,

frequency = ... Hz

2. using the graph, or otherwise, state the amplitude.

amplitude = ... cm [2]

14 The needle of a sewing machine is made to oscillate vertically through a total distance of 22 mm, as shown in Fig. 3.1.

For Examiner's Use

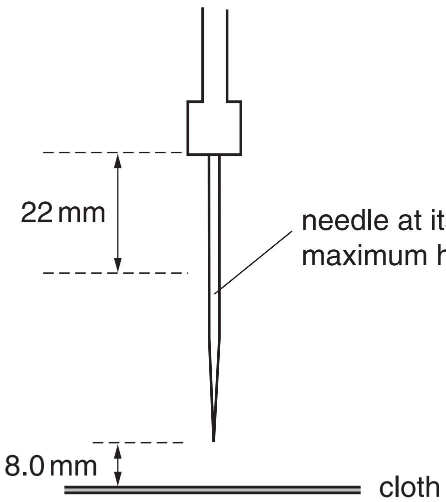


Fig. 3.1

The oscillations are simple harmonic with a frequency of 4.5 Hz.
The cloth that is being sewn is positioned 8.0 mm below the point of the needle when the needle is at its maximum height.

(a) State what is meant by *simple harmonic motion*.

....

....

.... [2]

(b) The displacement y of the point of the needle may be represented by the equation

$$y = a \cos \omega t.$$

(i) Suggest the position of the point of the needle at time $t = 0$.

.... [1]

(ii) Determine the values of

1. a,

a = ... mm [1]

2. ω.

ω = ... rad s^{-1} [2]

(c) Calculate, for the point of the needle,

(i) its maximum speed,

speed = ... $m\,s^{-1}$ [2]

(ii) its speed as it moves downwards through the cloth.

speed = ... $m\,s^{-1}$ [3]

15 The variation with time t of the displacement x of the cone of a loudspeaker is shown in Fig. 4.1.

For Examiner's Use

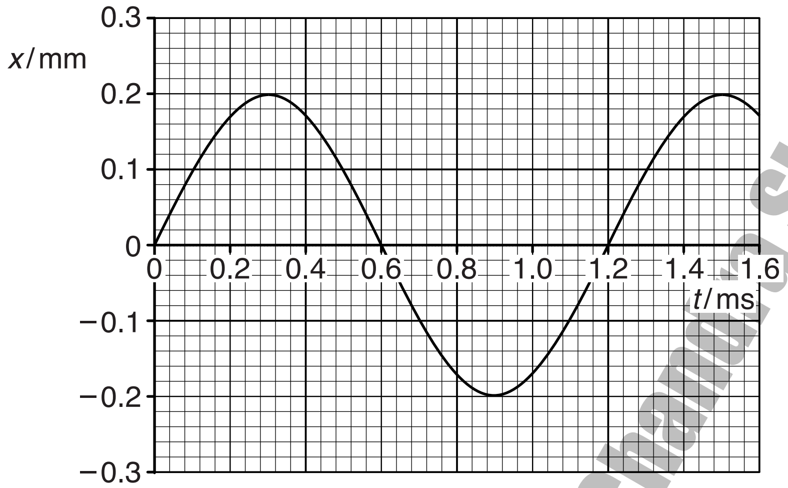


Fig. 4.1

(a) Use Fig. 4.1 to determine, for these oscillations,

(i) the amplitude,

amplitude = ... mm [1]

(ii) the frequency.

frequency = ... Hz [2]

(b) State two times at which

(i) the speed of the cone is maximum,

time .... ms and time .... ms [1]

(ii) the acceleration of the cone is maximum.

time .... ms and time .... ms [1]

(c) The effective mass of the cone is 2.5 g.

Use your answers in **(a)** to determine the maximum kinetic energy of the cone.

kinetic energy = ... J [3]

(d) The loudspeaker must be designed so that resonance of the cone is avoided.

(i) State what is meant by *resonance*.

....

....

.... [2]

(ii) State and briefly explain one other situation in which resonance should be avoided.

....

....

....

.... [2]

16 The variation with displacement x of the acceleration a of the centre of the cone of a loudspeaker is shown in Fig. 3.1.

For Examiner's Use

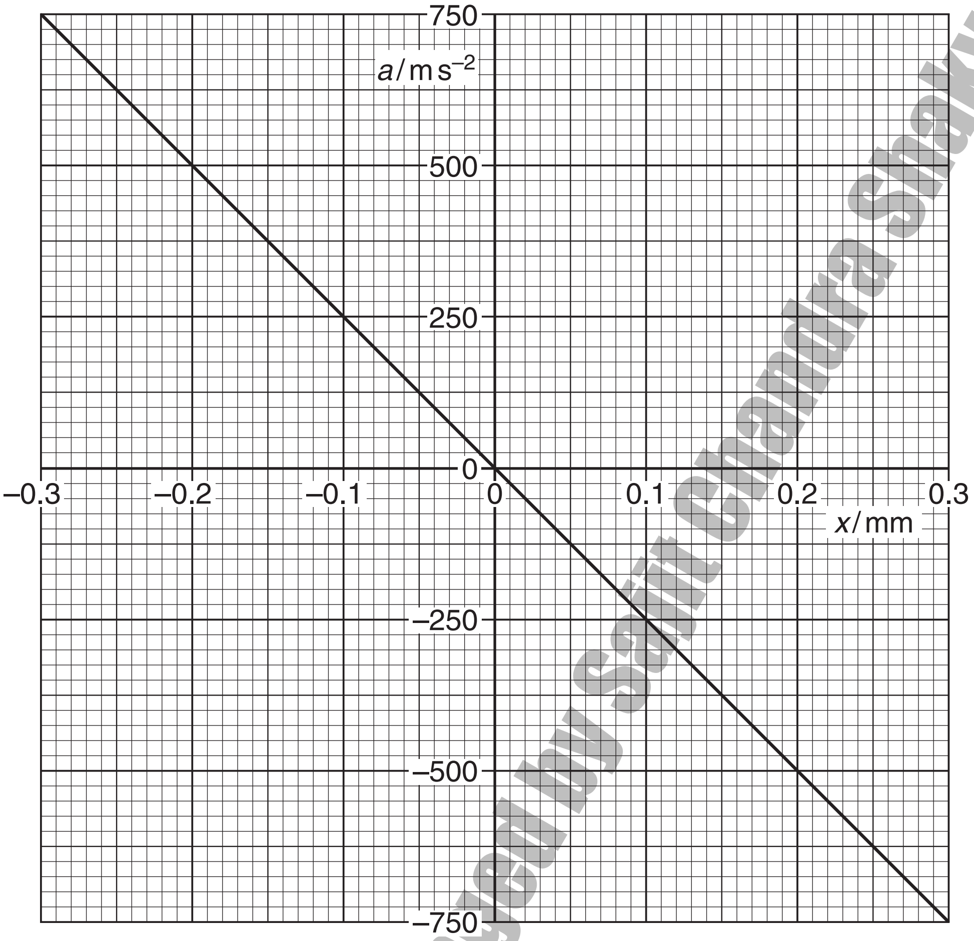


Fig. 3.1

(a) State the two features of Fig. 3.1 that show that the motion of the cone is simple harmonic.

1. ....

2. ....

[2]

(b) Use data from Fig. 3.1 to determine the frequency, in hertz, of vibration of the cone.

frequency = ... Hz [3]

For Examiner's Use

(c) The frequency of vibration of the cone is now reduced to one half of that calculated in **(b)**.

The amplitude of vibration remains unchanged.

On the axes of Fig. 3.1, draw a line to represent the variation with displacement x of the acceleration a of the centre of the loudspeaker cone.

[2]

17 A long strip of springy steel is clamped at one end so that the strip is vertical. A mass of 65 g is attached to the free end of the strip, as shown in Fig. 2.1.

For Examiner's Use

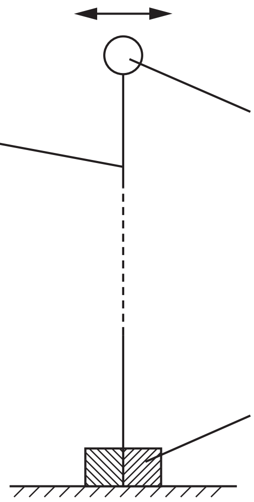


Fig. 2.1

The mass is pulled to one side and then released. The variation with time t of the horizontal displacement of the mass is shown in Fig. 2.2.

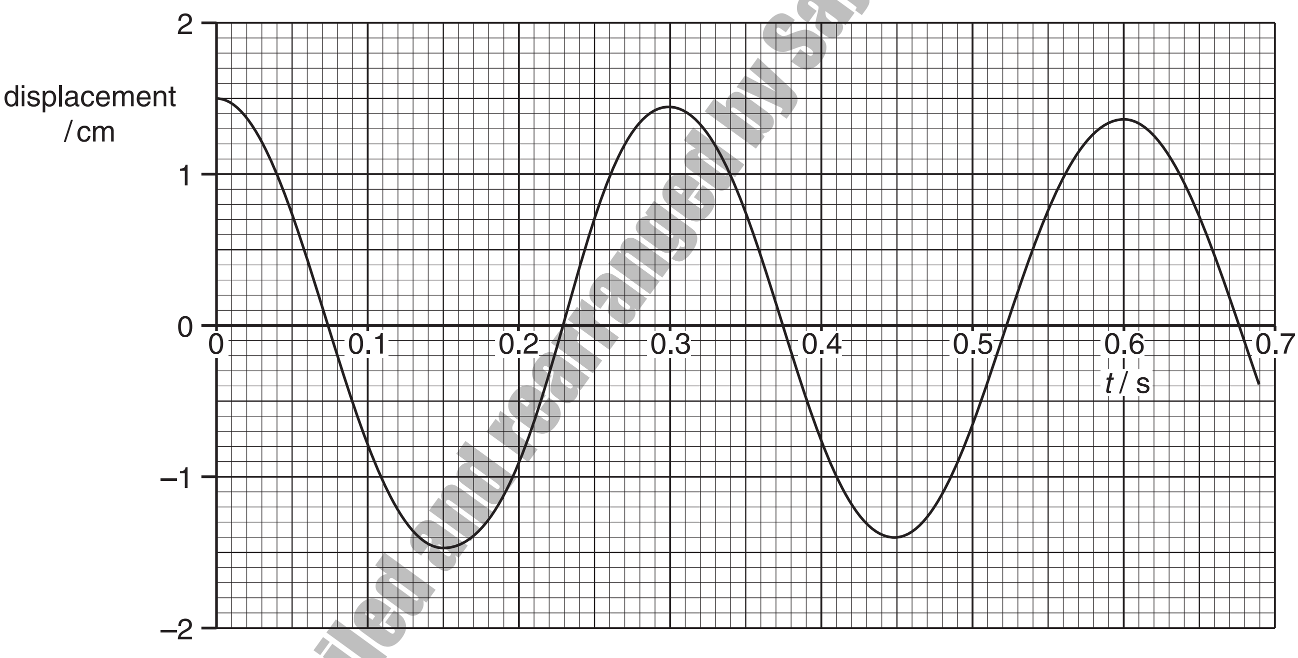


Fig. 2.2

The mass undergoes damped simple harmonic motion.

(a) **(i)** Explain what is meant by *damping*.

....

....

.... [2]

(ii) Suggest, with a reason, whether the damping is light, critical or heavy.

....

....

.... [2]

(b) (i) Use Fig. 2.2 to determine the frequency of vibration of the mass.

frequency = ... Hz [1]

(ii) Hence show that the initial energy stored in the steel strip before the mass is released is approximately 3.2 mJ.

[2]

(c) After eight complete oscillations of the mass, the amplitude of vibration is reduced from 1.5 cm to 1.1 cm. State and explain whether, after a further eight complete oscillations, the amplitude will be 0.7 cm.

....

....

.... [2]

18 **(a)** State what is meant by

(i) *oscillations*,

....

.... [1]

(ii) *free* oscillations,

....

.... [1]

(iii) *simple harmonic motion*.

....

....

.... [2]

(b) Two inclined planes RA and LA each have the same constant gradient. They meet at their lower edges, as shown in Fig. 3.1.

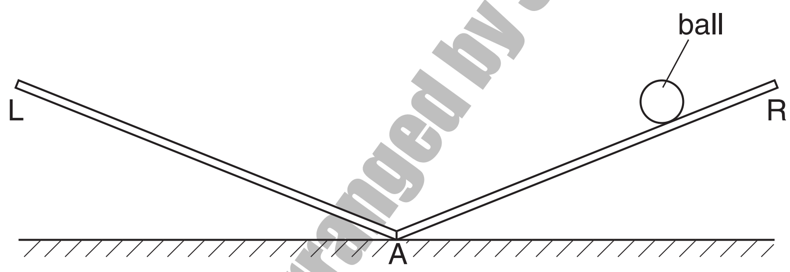


Fig. 3.1

A small ball moves from rest down plane RA and then rises up plane LA. It then moves down plane LA and rises up plane RA to its original height. The motion repeats itself.

State and explain whether the motion of the ball is simple harmonic.

....

....

.... [2]

3 A student sets up the apparatus illustrated in Fig. 3.1 in order to investigate the oscillations of a metal cube suspended on a spring.

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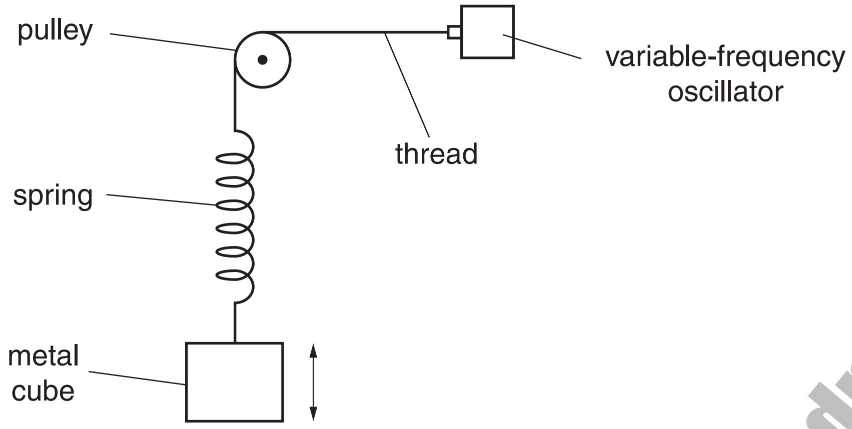

Fig. 3.1

The amplitude of the vibrations produced by the oscillator is constant.
The variation with frequency of the amplitude of the oscillations of the metal cube is shown in Fig. 3.2.

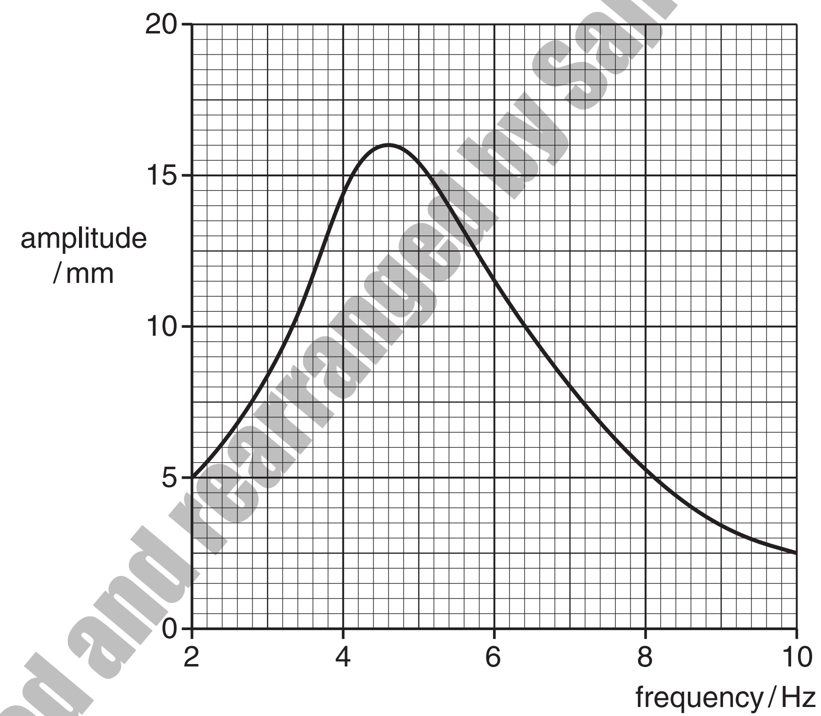

Fig. 3.2

(a) (i) State the phenomenon illustrated in Fig. 3.2.

.... [1]

(ii) For the maximum amplitude of vibration, state the magnitudes of the amplitude and the frequency.

amplitude = ... mm

frequency = ... Hz
[1]

(b) The oscillations of the metal cube of mass 150 g may be assumed to be simple harmonic.
Use your answers in **(a)(ii)** to determine, for the metal cube,

(i) its maximum acceleration,

acceleration = ... $m\,s^{-2}$ [3]

(ii) the maximum resultant force on the cube.

force = ... N [2]

(c) Some very light feathers are attached to the top surface of the cube so that the feathers extend outwards, beyond the vertical sides of the cube.
The investigation is now repeated.
On Fig. 3.2, draw a line to show the new variation with frequency of the amplitude of vibration for frequencies between 2 Hz and 10 Hz. [2]

3 A cylinder and piston, used in a car engine, are illustrated in Fig. 3.1.

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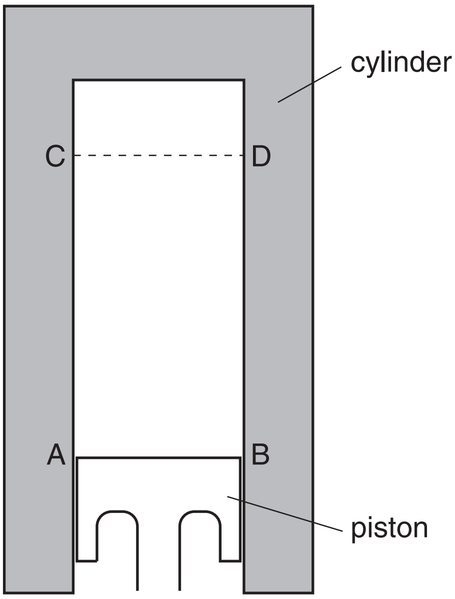

Fig. 3.1

The vertical motion of the piston in the cylinder is assumed to be simple harmonic.
The top surface of the piston is at AB when it is at its lowest position; it is at CD when at its highest position, as marked in Fig. 3.1.

(a) The displacement d of the piston may be represented by the equation

$$d = -4.0 \cos(220t)$$

where d is measured in centimetres.

(i) State the distance between the lowest position AB and the highest position CD of the top surface of the piston.

distance = ... cm [1]

(ii) Determine the number of oscillations made per second by the piston.

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number = ... [2]

(iii) On Fig. 3.1, draw a line to represent the top surface of the piston in the position where the speed of the piston is maximum. [1]

(iv) Calculate the maximum speed of the piston.

speed = ... cm s^{-1} [2]

(b) The engine of a car has several cylinders. Three of these cylinders are shown in Fig. 3.2.

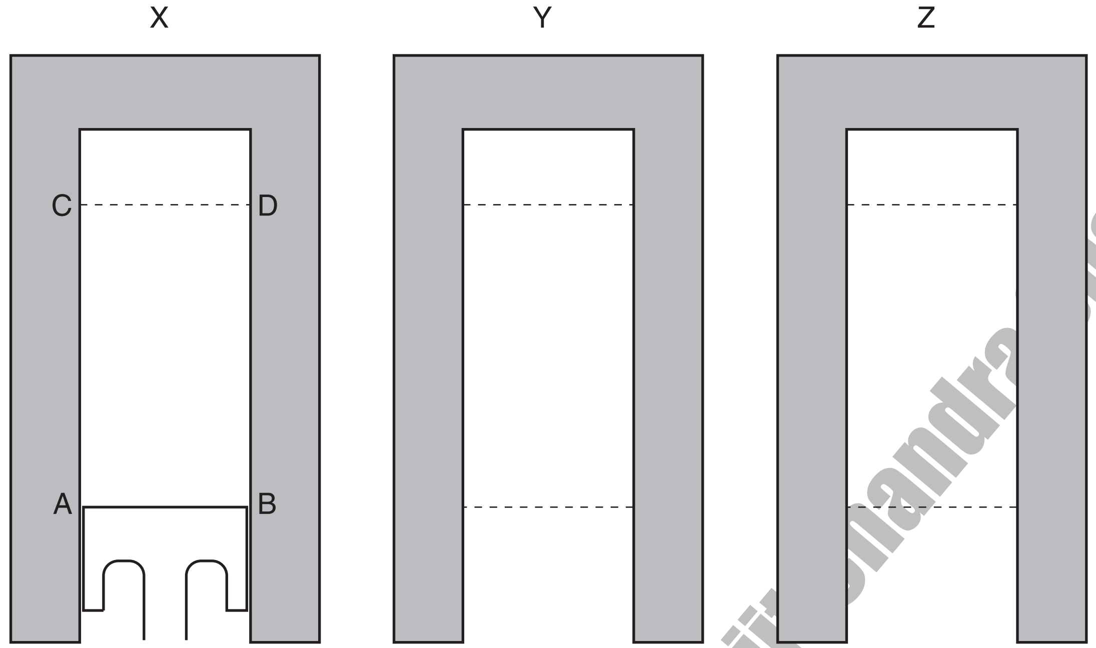

Fig. 3.2

X is the same cylinder and piston as in Fig. 3.1.
Y and Z are two further cylinders, with the lowest and the highest positions of the top surface of each piston indicated.
The pistons in the cylinders each have the same frequency of oscillation, but they are not in phase.
At a particular instant in time, the position of the top of the piston in cylinder X is as shown.

(i) In cylinder Y, the oscillations of the piston lead those of the piston in cylinder X by a phase angle of 120° ($\frac{2}{3}\pi$ rad).
Complete the diagram of cylinder Y, for this instant, by drawing

1. a line to show the top surface of the piston, [1]

2. an arrow to show the direction of movement of the piston. [1]

(ii) In cylinder Z, the oscillations of the piston lead those of the piston in cylinder X by a phase angle of 240° ($\frac{4}{3}\pi$ rad).

Complete the diagram of cylinder Z, for this instant, by drawing

1. a line to show the top surface of the piston, [1]

2. an arrow to show the direction of movement of the piston. [1]

(iii) For the piston in cylinder Y, calculate its speed for this instant.

speed = ... cm s^{-1} [2]

5 A bar magnet is suspended vertically from the free end of a helical spring, as shown in Fig. 5.1.

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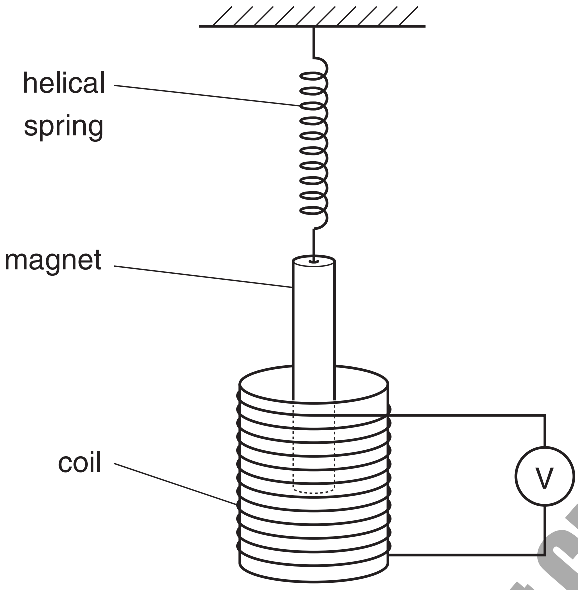

Fig. 5.1

One pole of the magnet is situated in a coil. The coil is connected in series with a high-resistance voltmeter.
The magnet is displaced vertically and then released.
The variation with time t of the reading V of the voltmeter is shown in Fig. 5.2.

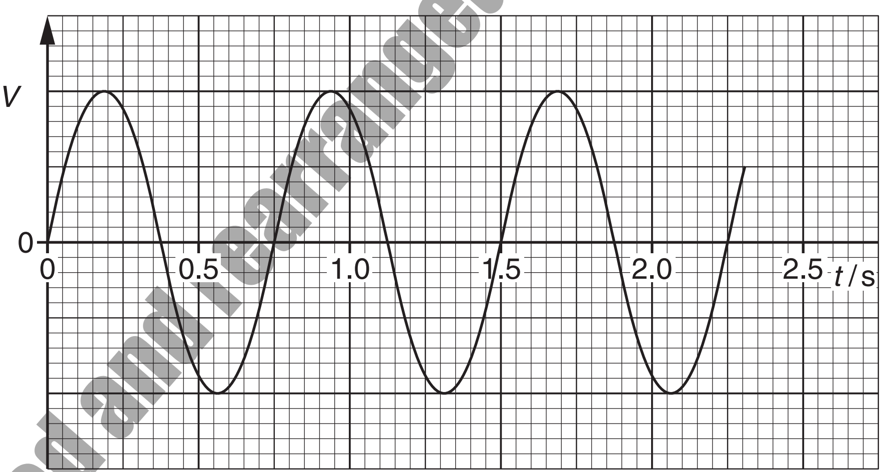

Fig. 5.2

(a) **(i)** State Faraday's law of electromagnetic induction.

....

....

.... [2]

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(ii) Use Faraday's law to explain why

1. there is a reading on the voltmeter,

....

.... [1]

2. this reading varies in magnitude,

....

.... [1]

3. the reading has both positive and negative values.

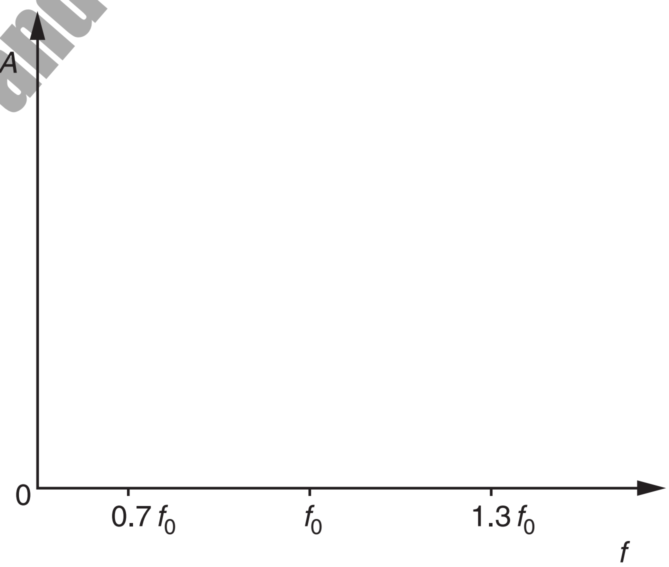
....

.... [1]

(b) Use Fig. 5.2 to determine the frequency f_0 of the oscillations of the magnet.

f_0 = ... Hz [2]

(c) The magnet is now brought to rest and the voltmeter is replaced by a variable frequency alternating current supply that produces a constant r.m.s. current in the coil.
The frequency of the supply is gradually increased from $0.7\,f_0$ to $1.3\,f_0$, where f_0 is the frequency calculated in **(b)**.
On the axes of Fig. 5.3, sketch a graph to show the variation with frequency f of the amplitude A of the new oscillations of the bar magnet.



[2]

Fig. 5.3

(d) (i) Name the phenomenon illustrated on your completed graph of Fig. 5.3.

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.... [1]

(ii) State one situation where the phenomenon named in **(i)** is useful.

....

.... [1]

3 **(a)** Define *simple harmonic motion*.

....

....

.... [2]

(b) A tube, sealed at one end, has a total mass m and a uniform area of cross-section A. The tube floats upright in a liquid of density ρ with length L submerged, as shown in Fig. 3.1a.

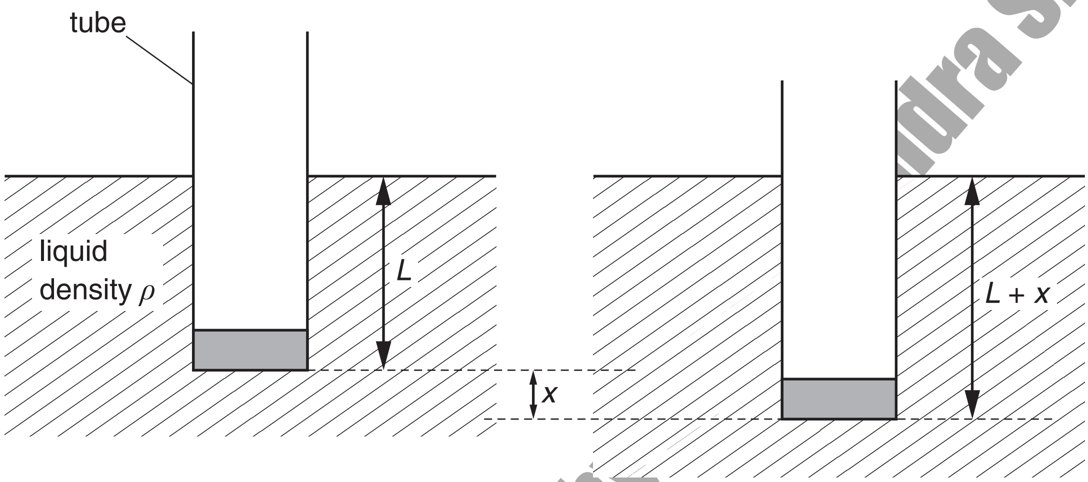

Fig. 3.1a **Fig. 3.1b**

The tube is displaced vertically and then released. The tube oscillates vertically in the liquid.
At one time, the displacement is x, as shown in Fig. 3.1b.

Theory shows that the acceleration a of the tube is given by the expression

$$a = -\frac{A\rho g}{m}x.$$

(i) Explain how it can be deduced from the expression that the tube is moving with simple harmonic motion.

....

....

....[2]

(ii) The tube, of area of cross-section $4.5\,cm^2$, is floating in water of density $1.0 \times 10^3\,kg\,m^{-3}$.

Calculate the mass of the tube that would give rise to oscillations of frequency 1.5 Hz.

mass = ... g [4]

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